TB146610A **Reg. No………………………….**

Name…………………………....

B. Sc. DEGREE (C.B.C.S.S.) EXAMINATION, MARCH 2017

SEMESTER VI – MATHEMATICS

MAT6RA – REAL ANALYSIS

Time: Three Hours **Maximum Marks: 80**

PART A

I. Answer all questions. Each question carries 1 mark.

1. State Cauchy's General principle of Convergence for series
2. Define Absolute convergence
3. State Rabee's Test
4. State Darboux's Theorem
5. Define Uniform convergence of a sequence of functions
6. Is the function f where $f(x) = \frac{x-|x|}{x}$ continuous?
7. Define discontinuity of the second kind
8. State Intermediate value theorem
9. For any two partitions P_1, P_2. Show that $L(P_1, f) \leq U(P_2, f)$
10. If $f \in \mathcal{R}[a,b]$, prove that $m(b-a) \leq \int_a^b f dx \leq M(b-a) where\ \ m\ and\ M$ are the bounds of $f\ on\ [a,b]$

(10×1 = 10)

PART B

II. Answer any eight questions. Each question carries 2 marks.

11. Test for convergence the series whose nth term is $\left\{(n^3+1)^{1/3} - n\right\}$
12. Show that the series $\sum \frac{(-1)^{n+1}}{\sqrt{n}}$ is conditionally convergent
13. If a refinement P^* of P contains p points more than P and $|f(x)| \leq k$ for all $x \in [a,b]$. Prove that $L(P^*, f) \leq L(P, f) + 2pk\mu$, where μ is norm of P.
14. Prove that if a function f is monotonic on [a, b], then it is Integrable on [a, b]
15. Show that the function [x], where [x] denotes the greatest integer not greater than x, is integrable on [0, 3] and $\int_0^3 [x] dx = 3$
16. Show that the series $\sum \frac{(-1)^n}{n} |x|^n$ is uniformly convergent in $-1 \leq x \leq 1$
17. Show that the function $f(x) = \frac{1}{1+|x|}$ for real x is continuous and bounded and attains its supremum for $x = 0$ but does not attain the infimum
18. Prove that if a function is continuous in a closed interval, then it is bounded therein
19. State and prove Fixed point theorem
20. Show that the function $f(x) = \frac{1}{x}$ is not uniformly continuous on (0, 1]

21. Show that $\sum \frac{a_n}{n^x}$ converges uniformly in [0, 1], if $\sum a_n$ converges
22. State Dirichlet's Test

(8 ×2 =16)

PART C

III. Answer any six questions. Each question carries 4 marks.

23. Show that the series $\sum \frac{3.6.9...3n}{7.10.13...(3n+4)} x^n, x > 0$ converges for $x \leq 1$ *and diverges for* $x > 1$
24. Prove that a function which is continuous on a closed interval is also uniformly continuous on that interval
25. Show that the function $f(x)$ defined on R by $f(x) = \begin{cases} x, & when\ x\ is\ irrational \\ -x, & when\ x\ is\ rational \end{cases}$ is continuous only at $x = 0$
26. Prove that $f(x) = \sin x^2$ is not uniformly continuous on $[0, \infty)$
27. If f is bounded and integrable on [a, b]. Then prove that $|f|$ is also bounded and integrable on [a, b], moreover $\left|\int_a^b f\,dx\right| \leq \int_a^b |f|\,dx$
28. Prove that a bounded function f, having a finite number of points of discontinuity on [a, b] is integrable on [a, b]
29. State and prove the fundamental theorem of calculus
30. Show that the sequence $\{f_n\}$ *where* $f_n(x) = \frac{x}{1+nx^2}, x\ real$ converges uniformly on any closed interval I
31. State and prove Cauchy's Criterion for uniform convergence for series

(6 × 4 =24)

PART D

IV. Answer any two questions. Each question carries 15 marks.

32. State and prove Cauchy's Root test. Test for convergence the series whose general term is $\left(1 + \frac{1}{\sqrt{n}}\right)^{-n^{3/2}}$
33. If a function f is continuous on a closed interval [a, b] and $f(a)$ *and* $f(b)$ are of opposite sign, then there exist at least one point $\alpha \in (a, b)$ such that $f(\alpha) = 0$
34. If a function f is bounded and integrable on [a, b]. Prove that the function F defined as $F(x) = \int_a^x f(t)dt,\ a \leq x \leq b$ is continuous on [a, b] and furthermore, if f is continuous at a point c of [a, b], then F is derivable at c and $F'(c) = f(c)$
35. State and prove Weierstrass M-Test. Show that the series $\sum \frac{\cos n\theta}{n^p}$ is uniformly and absolutely convergent for all real x if p >1

(2 ×15 = 30)