**TB146640A** **Reg. No…………………………….**

**Name…………………………......**

**B. Sc. DEGREE (C.B.C.S.S.) EXAMINATION, MARCH 2017**

**SEMESTER VI - MATHEMATICS**

**MAT6LA - LINEAR ALGEBRA AND METRIC SPACES**

**Time: Three Hours** **Maximum Marks: 80**

## PART A

**I. Answer all questions. Each question carries 1 mark.**

1. Give an example for a linearly independent set in $M_{2X2}(R)$.
2. Prove that, "A set of vectors in a vector space V that contains a zero vector is linearly dependent."
3. If a linear transformation $T: V \to W$ is one to one then nullity (T) = -------
4. If $T: P^2 \to M_{2X2}$ is defined by $T(at^2 + bt + c) = \begin{bmatrix} a & 2b \\ 0 & a \end{bmatrix} \forall\, a, b, c \in R$, then what is T(t) ?
5. If $T: R^2 \to R^2$ is a linear transformation with T [ a b ] = [ a 0 ] then $T^2$ [ ab ] = ------ , $\forall$a,b $\in R$
6. Define a metric space.
7. Give an equivalent condition for a set A to be a closed set.
8. Give an example of a subset of the real line which is neither open nor closed.
9. Give an example of a function on the set of real numbers which is continuous but not uniformly continuous.
10. State Baire's Theorem.

**(10x1=10)**

## PART B

**II. Answer any eight questions. Each question carries 2 marks.**

11. Determine whether $B = \{ f_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, f_2 = \begin{bmatrix} 1 \\ -1 \end{bmatrix} \}$ is a basis for $R^2$.
12. Define Projection onto the x axis and check whether it is linear or not?
13. Show that the kernel of a linear transformation is a subspace of the domain.
14. Prove that if L: $R^n \to R^m$ is defined as L(**u**) = A**u** for an mxn matrix A, then L is linear.
15. Find the transition matrix from first listed basis to the second $C = \left\{ \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \end{bmatrix} \right\}, D = \{ \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \end{bmatrix} \}$
16. Prove that in any metric space X, the empty set and the full space are open sets.
17. Prove that in any metric space X, each open sphere is an open set.
18. Prove or disprove every Cauchy sequence in a metric space is convergent.
19. Define continuity at a point in a metric space X.
20. Let X and Y be metric spaces and f a continuous mapping of X into Y. Prove that for a sequence $\{x_n\}$ converging to $x_0$, the sequence $\{f(x_n)\}$ converges to $f(x_0)$.
21. Give the difference between limit and limit point with example.

22. Let X be a complete metric space and let Y be a subspace of X. If Y is closed, then prove that it is complete.

**(8x2=16)**

## PART C

**III. Answer any six questions. Each question carries 4 marks.**

23. Prove that a finite set of vectors is linearly dependent iff one of the vectors is the linear combination of the vectors that precede it in the ordering established by the listing of the vectors in the set.

24. Let $T: R^2 \to R^3$ defined by $T\binom{a}{b} = \begin{bmatrix} a+b \\ a-b \\ 2b \end{bmatrix}$. Find the matrix representation of T with respect to the bases B = $\left\{\begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \end{bmatrix}\right\}$ for $R^2$ and C = $\left\{\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}\right\}$ for $R^3$.

25. Determine whether the linear transformation $T: R^2 \to R^3$ defined by $T\binom{a}{b} = \begin{bmatrix} a+b \\ a-b \\ 2a+3b \end{bmatrix}$ is one-one.

26. If S and T be 2 linear transformations from V to W and prove that both ST and S+T are linear.

27. Let X be a metric space with metric d. Show that $d_1$ defined by $d_1(x,y) = \frac{d(x,y)}{1+d(x,y)}$ is also a metric on X.

28. Let X be a metric space. Prove that a subset F of X is closed if and only if its complement F' is open.

29. Let X be a complete metric space and Y be a subspace of X. Then prove that Y is complete if and only if it is closed.

30. Prove that if a convergent sequence in a metric space has infinitely many distinct points, then its limit is a limit point of the set of points of the sequence.

31. Let X and Y be metric spaces and f a mapping of X into Y. Then prove that f is continuous if and only if $f^{-1}(G)$ is open in X whenever G is open in Y.

**(6x4=24)**

## PART D

**IV. Answer any two questions. Each question carries 15 marks.**

32. Prove that, Let U be a subspace of $R^3$
    (i) If dim U = 0, then U contains just the origin.
    (ii) If dim U = 1, then the graph of U is a straight line passing through the origin.
    (iii) If dim U = 2, then the graph of U is a plane that includes the origin.

33. State and prove Rank Nullity theorem.

34. Let X be a metric space. Prove the following :
    (i) Any intersection of closed sets in X is closed.
    (ii) Any finite union of closed sets in X is closed.

35. (i) State and prove Cantor's Intersection theorem
    (ii)Construct cantor set.

**(2x15=30)**