TB146620A **Reg. No………………………….**

Name…………………………....

B. Sc. DEGREE (C.B.C.S.S.) EXAMINATION, MARCH 2017

SEMESTER VI - MATHEMATICS

MAT6CA - COMPLEX ANALYSIS

Time: Three Hours **Maximum Marks: 80**

PART A

I. Answer all questions. Each question carries 1 mark.

1. Write an example for an entire function.
2. Write the domain of definition of the function $f(z) = \frac{1}{z^2+1}$
3. Show that the function $u = y^3 - 3x^2y$ is harmonic.
4. Define analytic function.
5. Write the function $f(z) = z^2 + z + 1$ in the form $f(z) = u(x, y) + i\, v(x, y)$
6. Define harmonic function
7. Evaluate $\int_0^{\frac{\pi}{6}} e^{i2t}$
8. State Cauchy-Goursat theorem.
9. When the arc c is said to be simple.
10. Evaluate $\int_1^2 (\frac{1}{t} - i)^2\, dt$

(10x1=10)

PART B

II. Answer any eight questions. Each question carries 2 marks.

11. Show that $f(z) = \bar{z}$ is nowhere differentiable.
12. Show that if $\lim_{z\to z_0} f(z)$exists, then it is unique.
13. Show that $Log\ (1 + i)^2 = 2\, Log\ (1 + i)\ but\ \ Log\ (-1 + i)^2 \neq 2\, Log\ (-1 + i)$.
14. Show that if $f'(z) = 0$ in a domain D then f(z) is a constant throughout D.
15. Show that if $f(z) = e^{-y} \sin x - ie^{-y} \cos x$ then it is an entire function.
16. Show that if v and V are harmonic conjugate of u(x,y) in a domain D, then v(x,y) and V(x,y) can differ atmost by an additive constant,
17. Show that if m and n are integers $\int_0^{2\pi} e^{im\theta} e^{-in\theta} d\theta = \begin{cases} 0 & when\ m \neq n \\ 2\pi & when\ m = n \end{cases}$
18. Evaluate $\int_c \frac{e^z}{z-1} dz$ where c is the circle $|z| = 2$in the positive direction.
19. Define simply connected domain and multiply connected domain with an example.
20. Let C be any simple closed contour, described in the positive sense in the z plane, and $g(z) = \int_c \frac{s^3+2s}{(s-3)^3} ds$, Show that $g(z) = 6\pi iz$ when z is inside C
21. Evaluate the integral $\int_1^2 (\frac{1}{t} - i)^2 dt$

22. Find the value of $\int_C \frac{ze^z}{(z^2+9)^5} dz, where\ c: |z| = 2$

(8x2=16)

PART C

III. Answer any six questions. Each question carries 4 marks.

23. If $limit_{z \to z_0} f(z) = w_0$ and $limit_{z \to z_0} g(z) = W_0$, then show that
$\text{limit}_{z \to z_0} f(z)g(z) = w_0 W_0$
$\text{limit}_{z \to z_0} \frac{f(z)}{g(z)} = \frac{w_0}{W_0}$
24. A function f(z) = u+iv is analytic in a domain D iff v is a harmonic conjugate of u.
25. Show that u is a harmonic function and find a harmonic conjugate v where u = $\frac{y}{x^2+y^2}$
26. If f(z) = u + iv and $z_0 = x_0 + iy_0$ then $\lim_{(x,y)\to(x_0,y_0)} u(x,y) = u_0$ and
$\lim_{(x,y)\to(x_0,y_0)} v(x,y)$
27. Evaluate $\int_C f(z)dz$ where $f(z) = \frac{z+2}{z}$ and c is the semicircle $z=2e^{i\theta}$ $(\pi \le \theta \le 2\pi)$
28. State and prove Maximum modulus principle.
29. State and prove Fundamental theorem of Algebra.
30. Evaluate $\int_C \frac{1}{z^2+4} dz$ where c is the circle $|z - i| = 2$ in the positive sense.
31. Give two Laurent series expansions in power series of z for the function f(z) = $\frac{1}{z^2\,(1-z)}$ and specify the regions in which those expansions are valid .

(6x4=24)

PART D

IV. Answer any two questions. Each question carries 15 marks.

32. State and prove necessary and sufficient condition for $f(z) = u + iv$ to be analytic.
33. a) If a function $f(z) = u + iv$ is analytic in a domain D then its component function u and v are harmonic in D.
b) State and prove chain rule for composite function
34. a) Evaluate $\int_C \frac{zdz}{(9-z^2)(z+i)}$ where c is the circle $|z| = 2$
b) Without evaluating the integral, S.T $\left|\int_C \frac{dz}{z^2-1}\right| \le \frac{\pi}{3}$ where c is the arc of the circle from to that lies in the 1st quadrant.
35. a) State and prove Cauchy integral formula.
b) State and prove Louville's theorem.

(2x15=30)