TB142410A **Reg.No………………..**

Name………………….

B.Sc. DEGREE (CBCSS) EXAMINATION, APRIL 2015
SECOND SEMESTER – COMPLEMENTARY COURSE (STATISTICS)
STA2TRV – THEORY OF RANDOM VARIABLES
(COMMON FOR MATHEMATICS, PHYSICS & COMUTER APPLICATIONS)

Time: 3 hours **Max: 80 marks**

Use of Scientific calculators and Statistical tables are permitted

Part A
(Short Answer Questions)
Answer *all* questions. Each question carries 1 mark

1. Define a random variable.
2. What are the properties of a probability density function?
3. State the condition for independence of two random variables.
4. Define characteristic function of a random variable.
5. If *X* is a random variable, show that $E(X^2) \geq \left[E(X)\right]^2$.
6. Find the moment generating function of a random variable *X* with probability density function $f(x) = 1,\ 0 < x < 1$.
7. Define skewness.
8. Give the formula for computing Spearman’s rank correlation coefficient.
9. What would be your interpretation if the correlation coefficient is 0?
10. If the variables are independent what will be the angle between the regression lines?

(10x1 = 10 marks)

Part B
(Brief Answer Questions)
Answer any *eight* questions. Each question carries 2 marks

11. Define distribution function of a random variable and state its properties.
12. For the *p.m.f*, $f(x) = C\left(\frac{1}{2}\right)^x$, $x = 0, 1, 2, \ldots$, evaluate the constant *C*.
13. Given that $f(x) = 3x^2;\ 0 < x < 1$ is a *p.d.f.* Determine the *p.d.f* of $Y = X^2$.
14. State and prove addition theorem on Expectation of two random variables.
15. If *X* and *Y* are two independent random variables with characteristic functions $\phi_X(t)$ and $\phi_Y(t)$ respectively, show that $\phi_{X+Y}(t) = \phi_X(t)\phi_Y(t)$.
16. If X is a random variable with *p.m.f*, $f(x) = x/6;$ for $x = 1, 2, 3$. Find the mean and variance of *X*.
17. Define raw and central moments.
18. Define Kurtosis. What are the types of kurtosis?
19. What is a scatter diagram? How is it constructed?
20. Show that $r_{xy}^2 = b_{xy} \times b_{yx}$.
21. Karl Pearson’s coefficient of correlation of two variables *x* and *y* is 0.8. Their covariance is 40. If the variance of *x* is 16, find the standard deviation of *y*.
22. Write the normal equations for fitting a parabola $y = a + bx + cx^2$.

(8x2 = 16 marks)

Part C
(Short Essay Questions)
Answer any *six* questions. Each question carries 4 marks

23. The joint probability mass function of a bivariate r.v (*X*,*Y*) is given by: $f(x,y)=\dfrac{x+y}{18}$, $x=0,1,2$ and $y=0,1,2$. Find marginal distributions of *X* and *Y*.
24. Examine whether the variables *X* and *Y* are independent if the joint *p.d.f* of (*X*,*Y*) is given by $f(x,y)=6(x-y);\ 0<y<x<1$.
25. Find the covariance between *X* and *Y* if $f(x,y)=x+y, 0\le x,y\le 1$.
26. If the m.g.f. of a random variable *X* is $M_X(t)=(1-t)^{-1}$, find the measure $_1$.
27. Find $E(X\mid Y)$ if the joint p.d.f is $f(x,y)=8xy,\ 0<x<y<1$.
28. Show that $_2>1$ for a discrete distribution.
29. Derive the expression for Spearman's rank correlation coefficient.
30. Find the Karl Pearson's coefficient of correlation for the following data

x	-2	-1	1	2
y	4	1	1	4

31. How will you fit a curve of the form $y=ae^{bx}$?

(6x4 = 24 marks)

Part D
(Essay Questions)
Answer any *two* questions. Each question carries 15 marks

32. Two random variables *X* and *Y* have the following joint density function: $f(x,y)=k(4-x-y)$, if $0\le x\le 2, 0\le y\le 2$. Find i) the value of *k*,ii) the marginal density functions, iii) Conditional density functions and iv) $E(X^2Y)$.
33. Examine the nature of skewness and kurtosis for the following data using the measures $_1$ and $_2$.

Class	0-10	10-20	20-30	30-40
Frequency	1	3	4	2

34. The following table consists of the test scores of 6 randomly selected students and the number of hours they studied for the test. Obtain the regression equation for the test score related to hours of preparation:

Hours of preparation (*x*)	2	10	4	6	8	9
Test score (*y*)	8	46	19	26	30	32

Estimate the test score of a student who studied 5 hours for the test.

35. Given that $8x-10y+66=0$ and $40x-18y-214=0$ are the regression lines, i) identify the regression lines and find (ii) the correlation coefficient, iii) the mean values of *x* and *y*, iv) the ratio of variances of *x* and *y*.

(2x15 = 30 marks)