28/10/25

TM253602B

Reg. No :.....

Name :.....

MASTER'S DEGREE (C.S.S) EXAMINATION, NOVEMBER 2025

2024 ADMISSIONS REGULAR

SEMESTER III - CORE COURSE

MT3C15TM20 - Graph Theory

Time : 3 Hours **Maximum Weight : 30**

Part A

I. Answer any Eight questions. Each question carries 1 weight **(8x1=8)**

1. If G and H are isomorphic graphs then show that each pair of corresponding vertices of G and H has same degree.
2. Show that line graph of K_n is regular of degree 2n-4. Draw the line graph of K_4.
3. Prove that a vertex v of a connected graph G with at least 3 vertices is a cut vertex of G if and only if there exist vertices u and w of G distinct from v such that v is on every u-w path.
4. For a simple connected graph G if n(G) = m(G) show that G is unicyclic.
5. Prove that a tree with at least 2 vertices contains at least two pendant vertices.
6. Write a short note on Hamilton's 'Around the World' Game.
7. If G is k-regular prove that $\chi(G) \geq n/n-k$.
8. Write a short note on Jordan Curve Theorem.
9. Prove that a simple planar graph has at least 4 vertices of degree 5 at most.
10. Define (a) Dual of a planar graph (b) Canonical embedding

Part B

II. Answer any Six questions. Each question carries 2 weight **(6x2=12)**

11. Prove that if a simple graph G is not connected then its complement is connected.
12. Show that every vertex of a disconnected tournament T with $n \geq 3$ is contained in a directed k- cycle, where $3 \leq k \leq n$.
13. State and prove Whitney's Theorem.
14. Prove that a tree with n vertices have n-1 edges.
15. Let G be a simple graph with n≥3 vertices the G is Hamiltonian if and only if G + uv is Hamiltonian for all pair of non-adjacent vertices u and v in G with d(u)+d(v)≥n.
16. Define independence number. Prove that for any graph with n vertices and independence number α ;

$$\frac{n}{\alpha} \leq \chi \leq n - \alpha + 1.$$

17. a) Define Spherical embedding and Stereographic Projection. b) Prove that a graph is planar if and only if it is embeddable over a sphere.
18. Write a note on spectrum of graph.

Part C

III. Answer any Two questions. Each question carries 5 weight **(2x5=10)**

19. a) Define Bipartite graph. b) If a bipartite graph G is regular, show that |X| = |Y|. c) Let G be any non-empty graph with at least two vertices then show that G is bipartite if and only if it has no odd cycles.
20. Prove that $r(K_n)=n^{n-2}$; $n \geq 2$.

21. Prove that if a connected graph g is neither an odd cycle nor a complete graph then $\chi \leq \Delta(G)$.

22. a) Prove that K_5 and $K_{3,3}$ is non planar.

    b) State and prove Heawood five-color theorem.