22/10/25

TM251452U

Reg. No :.....

Name :.....

MASTER'S DEGREE (C.S.S) EXAMINATION, OCTOBER 2025

2025 ADMISSIONS REGULAR

SEMESTER I - CORE COURSE (MATHEMATICS)

MT1C03TM25 - Real Analysis

Time : 3 Hours **Maximum Weight : 30**

Part A

Answer any 2 questions from the bunch of CO1. Each question carries 1 weight. **(1x2=2)**

1. Explain functions of bounded variation. Check whether $f(x)=x^{1/3}$ is a function of bounded variation or not. [CO1,Analyse]
2. If f is monotonic on [a, b], establish that f is of bounded variation on [a, b]. [CO1,Analyse]
3. If f is monotonic on [a, b]. Illustrate that the set of discontinuities of f is countable. [CO1,Analyse]

Part A

Answer any 2 questions from the bunch of CO2. Each question carries 1 weight. **(1x2=2)**

4. Let $f_1 \in \mathcal{R}(\alpha)$ and $f_2 \in \mathcal{R}(\alpha)$ on [a, b]. If $f_1(x) \leq f_2(x)$ on [a, b], show that $\int_a^b f_1 \, d\alpha \leq \int_a^b f_2 \, d\alpha$. [CO2,Apply]
5. If f is continuous on [a, b] then $f \in \mathcal{R}(\alpha)$ on [a, b]. Justify. [CO2,Apply]
6. Show that $\int_a^b f \, d\alpha \leq \int_a^{\overline{b}} f \, d\alpha$ [CO2,Apply]

Part A

Answer any 2 questions from the bunch of CO3. Each question carries 1 weight. **(1x2=2)**

7. Analyse the definition of uniform convergence to check whether the function $f_n(x) = \frac{x}{1+nx^2} \; ; \; x \in [0, 1]$ converges uniformly or not. [CO3,Analyse]
8. Establish or disprove 'Limit of integral need not be equal to the integral of the limit even if both are finite.' [CO3,Analyse]
9. Establish that $f_n(x) = \frac{n}{x+n} \; ; x \geq 0, x \in [0, K]$ converges uniformly. [CO3,Analyse]

Part A

Answer any 2 questions from the bunch of CO4. Each question carries 1 weight. **(1x2=2)**

10. If $0 < t < 2\pi$, show that E(it)≠1. [CO4,Apply]
11. Verify that $\lim_{x \to 0}(1 + x)^{\frac{1}{x}} = e$ [CO4,Apply]
12. If $\{f_n\}$ is a uniformly bounded sequence of continuous functions on a compact set E, there need not exist a subsequence which converges pointwise on E. Justify this statement with an example. [CO4,Apply]

Part B

Answer any 3 questions from the bunch of CO3. Each question carries 2 weight. **(2x3=6)**

13. Explain Cauchy Criterion for Uniform Convergence. [CO3,Analyse]

14.

1. Suppose K is compact. $\{f_n\}$ is an increasing sequence of continuous functions on K which converges pointwise to a continuous function f on K. Establish that $f_n \to f$ uniformly on K.

[CO3,Analyse]

15. Explain Weirstrass M-test for uniform convergence. Check whether $f_n(x) = \frac{nx}{1+n^2x^2}$; $x \in [0,1]$ converges uniformly. [CO3,Analyse]

16. Suppose $f_n \in \mathcal{R}(\alpha)$ on [a, b] and if $f(x) = \sum_{n=1}^{\infty} f_n(x); a \le x \le b$, where the convergence is uniform, then show that $\int_a^b f \, d\alpha = \sum_{n=1}^{\infty} \int_a^b f_n \, d\alpha$. [CO3,Analyse]

Part B

Answer any 3 questions from the bunch of CO4. Each question carries 2 weight. **(2x3=6)**

17. If $\sum a_n, \sum b_n, \sum c_n$ converges to A, B, C and if $c_n = a_0 b_n + \ldots + a_n b_0$. Examine *C=AB.* [CO4,Apply]

18. If $\{f_n\}$ is a pointwise bounded sequence of complex functions on a countable set E, the $\{f_n\}$ has a subsequence $\{f_{n_k}\}$ such that $\{f_{n_k}(x)\}$ converges for every $x \in E$. [CO4,Apply]

19. Apply Stone Weierstrass theorem to establish that or every interval [-a, a], there is a sequence of real polynomials P_n such that $P_n(0)=0$ and $\lim_{n\to\infty} P_n(x) = |x|$ uniformly on [-a, a]. [CO4,Apply]

20. Explain Taylor's Theorem. [CO4,Apply]

Part C

Answer any 1 question from the bunch of CO1. Each question carries 5 weight. **(5x1=5)**

21. a) Assume that f and g are functions of bounded variation on [a, b]. Establish that their sum, difference and product are also functions of bounded variation on [a, b].
b) Show that $V_{f\pm g} \le V_f + V_g$ and $V_{f.g} \le AV_f + B V_g$, where A=sup {g(x) :x ∈ [a, b]} and B=sup {f(x) :x ∈ [a, b]} .
c) Is the quotient of function, a function of bounded variation? Justify your answer. [CO1,Analyse]

22. a) Let f be an increasing function defined on [a, b] and let $x_0, x_1, \ldots, x_n$ be n+1 points such that $a=x_0<x_1<\ldots<x_n=b$. Then establish the inequality $\sum_{k=1}^{n-1} [f(x_k+) - f(x_k-)] \le f(b) - f(a)$ b) Let f be a rectifiable path defined on [a, b], let $s(x)=\begin{cases} \Lambda_f(a,x) & ; \text{if } x \in (a,b] \\ 0 & ; \text{if } x = a \end{cases}$. Show that

i. The function s is continuous and increasing on [a, b].
ii. If there is no subinterval of [a, b] on which f is constant, then s is strictly increasing on [a, b].

[CO1,Analyse]

Part C

Answer any 1 question from the bunch of CO2. Each question carries 5 weight. **(5x1=5)**

23. a) Verify that $f \in \mathcal{R}(\alpha)$ on [a, b] if and only if $\forall \varepsilon > 0$ there exist a partition P such that $U(P,f,\alpha) - L(P,f,\alpha) < \varepsilon$. b) Suppose f is bounded on [a, b], f has only finitely many points of discontinuity on [a, b], and α is continuous at every point at which f is discontinuous. Then establish that $f \in \mathcal{R}(\alpha)$.

[CO2,Apply]

24. a) Let $f \in \mathcal{R}$ on [a, b]. For $a \le x \le b$, $F(x) = \int_a^x f(t)\,dt$. Establish that F is continuous on [a, b], furthermore if f is continuous at a point x_o of [a, b] then F is differentiable at x_o and $F'(x_o) = f(x_o)$. b) If $f \in \mathcal{R}$ on [a, b] and if there is a differential function F on [a, b] such that $F' = f$, then $\int_a^b f(x)dx = F(b) - F(a)$.

[CO2,Apply]