

B.Sc DEGREE (CBCSS) EXAMINATION, NOVEMBER 2014
FIRST SEMESTER - COMPLEMENTARY COURSE (MATHEMATICS)
MAT1DCT – DIFFERENTIAL CALCULUS AND TRIGONOMETRY
(COMMON FOR B. Sc PHYSICS AND CHEMISTRY)

Time: 3 hrs

Max. Marks: 80

PART A**(Short Answer questions)**(Answer **all** questions. Each question carries 1 mark)

1. Find $\lim_{x \rightarrow 1} \left(\frac{3-2x}{-1-x} \right)$
2. State Sandwich theorem.
3. Find $\frac{d}{dx} (\sin x \cos x)$
4. What is the absolute maximum of $f(x) = \frac{2}{3}x - 5, -2 \leq x \leq 3$.
5. Give an example of an increasing function in $[0, 1]$.
6. State the mean value theorem.
7. Find f_x if $f(x, y) = \frac{2y}{y + \cos x}$
8. If $w=f(x, y, z)$ find $\frac{dw}{dt}$ with the help of tree diagram.
9. State De Moivre's theorem.
10. Prove that $\cos 3\theta = 4 \cos^3 \theta - 3 \cos \theta$.

(10 x 1 = 10 marks)

PART B**(Brief Answer questions)**(Answer any **eight** questions. Each question carries 2 marks)

11. Evaluate $\lim_{x \rightarrow 1} \frac{x^2 + x - 2}{x^2 - x}$
12. Find $\lim_{x \rightarrow 0} u(x)$ if $1 - \frac{x^2}{4} \leq u(x) \leq 1 + \frac{x^2}{2}$.
13. Find the function $f(x)$ whose derivative is $\sin x$ and whose graph passes through $(0, 2)$.
14. Find c in the mean value theorem for $f(x) = x^2 + 2x - 1, [0, 1]$.
15. If $f(x, y) = 1 - x + y - 3x^2y$ find $\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}$ at $(1, 2)$.
16. Show that $f(x, y, z) = x^2 + y^2 - 2z^2$ satisfies the Laplace equation $\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2} = 0$.
17. Find dy/dx at $t = 6$ if $x = 2t + 3$ and $y = t^2 - 1$.
18. Which order of differentiation will calculate f_{xy} faster for $f(x, y) = x \sin y + e^y$. Why?
19. Show that $x^3 + 3x + 1 = 0$ has exactly one real solution.

(P.T.O)

20. Show that $\cos 4\theta = \cos^4 \theta - 6\cos^2 \theta \sin^2 \theta + \sin^4 \theta$.
21. Prove that $\cosh^2 y - \sinh^2 y = 1$.
22. Separate into real and imaginary parts, $\sinh(a+ib)$.

(8 x 2 = 16 marks)

PART C

(Descriptive Short Essay Questions)

Answer any **six** questions. Each question carries 4 marks

23. Find the slope of the circle $x^2 + y^2 = 25$ at the point (3,-4)
24. Find a parametrization for the line segment with end points (-2,1) and (3,5).
25. Find the absolute maximum and absolute minimum of $f(x) = x^{2/3}$ on [-2, 3].
26. If $f'(x) = 0$ at each point x of an interval (a, b) then show that $f(x)$ is a constant.
27. Define monotonic functions. Give examples. State the first derivative test for monotonic functions.
28. Find the extreme values of the function $y = 2x^2 - 8x + 9$ and where do they occur?
29. Expand $\sin^6 \theta$ in a series of cosines of multiples of θ .
30. Show that $\tanh 3x = \frac{3 \tanh x + \tanh^3 x}{1 + 3 \tanh^2 x}$
31. If $\sin(A + iB) = x + iy$, prove that $\frac{x^2}{\sin^2 A} - \frac{y^2}{\cos^2 A} = 1$

(6 x 4 = 24 marks)

PART D

(Essay type questions)

(Answer any **two** questions. Each question carries 15 marks)

32. (a) Differentiate implicitly to find dy/dx if $y = x^2 + \sin xy$
- (b) Show that the slope of every line tangent to the curve $y = 1/(1-2x)^3$ is positive.
- (c) Find the derivative of $\frac{t^2 - 1}{t^2 + 1}$
33. Find the critical points of $f(x) = x^{1/3}(x-4)$. Identify the intervals on which f is increasing and decreasing. Find the local and extreme values.
- 34.(a) If $h(x, y, z) = \dots \sin y \cos x$ find the partial derivatives with respect to each variable.
- (b) Find dw/dt if $w = xy + z$, $x = \cos t$, $y = \sin t$, $z = t$. What is the derivative's value at $t = 0$.
- 35.(a) Separate into real and imaginary parts $\tan^{-1}(a + ib)$.
- (b) Sum the series $\cosh r - \frac{1}{2} \cosh 2r + \frac{1}{3} \cosh 3r - \dots$

(2 x 15 = 30 marks)