Project Report

On

MATHEMATICS IN FORENSIC SCIENCE

Submitted

in partial fulfilment of the requirements for the degree of

BACHELOR OF SCIENCE

in

MATHEMATICS

by

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(Register No. AB22BMAT007)

Under the Supervision of

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THE FEAR OF THE LORD IS THE BEGINNING OF WISDOM

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DEPARTMENT OF MATHEMATICS

ST. TERESA'S COLLEGE (AUTONOMOUS)

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CERTIFICATE

This is to certify that the dissertation entitled, **MATHEMATICS IN FORENSIC SCIENCE** is a bonafide record of the work done by Ms. **ADITHYA C S** under my guidance as partial fulfilment of the award of the degree of **Bachelor of Science in Mathematics** at St. Teresa's College (Autonomous), Ernakulam affiliated to Mahatma Gandhi University, Kottayam. No part of this work has been submitted for any other degree elsewhere.

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CHAPTER 1

MATHEMATICS IN FORENSIC SCIENCE

Forensic science is the application of science to criminal and civil laws mainly on the criminal side during criminal investigation as governed by the legal standards of admissible evidence and criminal procedure.

Forensic science is an ever-growing field that can be further subdivided into Toxicology, Anthropology, and Odontology. Mathematics and many other applications of calculus of forensic science can be most clearly being seen in the fields of forensic Biology and pathology. For example, to put it in a simpler terms power series are to Functions what DNA molecules are to people.

Specifically for pathologists, calculus is needed to estimate the time of death of victims. Overall calculus has many applications to many of the subfield and forensics and is often a useful tool in crime scene investigation Forensic scientists collect preserve and analyze scientific evidence themselves. Others occupy a laboratory role performing analysis on objects brought to them by other individuals.

1.1 Measurement

One area of math is crucial to forensic science is taking precise measurements at a crime scene knowing the exact length of a shoe print could whose shoes are the wrong size. For example, forensic scientists need exact measurement of everything at a crime scene to perform scientific calculations properly. Investigators spend a great deal of time measuring distance, weight temperature volume and other aspects of evidence to get the number correct.

1.2 Proportions

Forensic scientists use not only measurements but proportion in other analysis. If a Human leg bone is discovered in a unmarked grave, for example forensic scientists use math equations to determine what proportion or percentage of a person overall height the leg bone would be. Once they know that they can determine how tall the person was and whether it was a child, or adult proportions are one way math is involved in forensic Science.

1.3 Application of different branch of math in criminal investigations

Forensic scientists analyze the existence in and around crime scenes for clues pointing to possible suspects causes of death or time of death. Math can be used to determine how crime is committed when they were committed and who committed and who committed them

- Psycho physical detection –Monitoring plus rate, blood pressure, and breathing Patterns
- Heights and distance – Footprints in dirt and mud length of strides
- Entomology-time of death
- Trigonometry and industrial physics can be used to reverse calculate height
- Examining the skid mark can help to reconstruct the accident mark that are

Caused by the speed of the car, breaking forces

- Newton’s law of cooling describes the cooling of a warmer objects to the cooler

Temperature of its environment

1.3.1 Calculus in forensic science

A forensic analyst uses blood stain pattern analysis in order to tell the story of the crime.

It turns out that the location where the blood lands and the shape of a blood on the landing surface reveal both the direction in which the blood was used to wound the victim. Analysis use math principles to figure out the location of the victim where the blood was shed and even the type of weapons or impact that caused the victims injury. Sometimes from a weapon can even reveal a criminals mentality A different kind of calculus is used by medical examiners and forensic technicians ever day to estimate time of death of a victim. When a victim dies, they experience three stages of death, where the body starts to decompose Algor mortis is the rate at which the body cools after death. Algor mortis is used to estimate time of death, importantly, if the body is discovered only a few hours or less after death, to determine the time of death by using Newton's law of cooling

$$\frac{dT}{dt} = K(T - T0)$$

where T is the temperature of the object, T0 is the temperature of the Surroundings.

1.3.2 Probability

Random math probabilities are used to estimate and express the rarity of a DNA that someone else in the population chosen at random would have the same genotype as the genotype of the contributor of the forensic evidence. RMP is calculated during the genotype frequencies at all the loci or how common or rare the alleles of a genotype are RMP can only be used as a statistic to describe the DNA profile if it is from a single source or if the analysis is able to differentiate between the peaks on the electropherogram from the major and minor contributors of a mixture.

Combined probability of inclusion is a common statistic used when the analyst cannot differentiate between the peaks from the major and minor contributor to a sample and the number of contributors cannot be determined

1.3.3 Trigonometry

Trigonometry is very useful in forensic science. Knowledge of trigonometry is necessary for many crimes scene reconstructions blood stain pattern analysis is one of the several specialists in the field of forensic science involves the study and analysis of blood stains at a known or suspected violent acts such as assault, homicide, abduction, suicide or even vehicular accidents. Pythagoras theorem, Trigonometry function, Trigonometry rules are applications of trigonometry in forensic science. Trigonometry functions relate to non-right-angled triangles and can be used to find a uniform angle or side.

1.4 Police mathematics

Police use mathematics every day on their job to solve crimes. Policemen jobs are to find out what happened at the sight of the scene of the crime or accident. They use math on the job to explain data that needs to be stored for information.

For example, a speeding car causing skid marks. Data can be stored and interpreted using wavelets, probability and statistics. It can be securely transmitted using prime numbers and cryptography. But first, police must get at the information underlying the data. They must look at all the evidence left at the crime scene and work backwards to deduce what happened and who did it. For the officer to find out how fast the car was going at the scene he needs to solve an inverse problem. Inverse problems are mathematical detective problems. An example of an inverse problem is trying to find the shape of an object only knowing its shadows. In addition, a day on the job of being a cop. There is a car accident, and the officer job is to figure out if the car was speeding or not. The only information the police officer has is the damage to the car, witnesses, and skid marks from the vehicle. By investigating the skid marks can help rebuild the accident. The marks are caused by the speed of the car as well as other factors such as braking force, friction with the road and impacts with other vehicles. Without any witnesses at the scene of the accident, the police officers must plug in a formula to figure out who is at fault of the accident.

CHAPTER 2

GEOMETRY IN FORENSIC SCIENCE

Geometry plays a crucial role in forensic science by helping investigators analyze crime scene and reconstruct events. Investigators use geometric principles to create accurate models of crime scene. This includes mapping the layout of a scene, determining angles of entry or exit, and analyzing the position of evidence

Blood stain pattern analysis

Blood stain pattern analysis is a forensic discipline that deals with the physics of blood and assess blood stain left at crime scene using visual pattern recognition. The shape and distribution of blood stain can reveal information about the events that occurred during a violent crime. Analysts used geometric formulas to determine the angle at which blood droplets fell and to reconstruct the movement of victims and assailants.

2.1 Blood Spatter:

When we speak of blood spatter, we mean the way that the blood is being distributed, the shape of the droplets when it hits the surface, and the angle of impact there are many types of blood spatter. Because blood doesn't always just drip out of the wound, we don't always have perfect droplets of blood. Instead, we get what looks like smears, a droplet followed by an elongation (tear drop shaped) and or many tiny little droplets distributed about the surface they landed on. The blood stain which is left is always bigger than the actual droplet; this is because the volume of blood is dispersed on the surface. When constructing a right triangle to determine at which angle the droplet fell, we must be certain that the angle outside the right triangle is equal to the right triangle on the inside. This way we can measure the bloodstain.

According to Yonder, Anita, when a blood drop appears to be tear drop shaped, it is several cross sections of the sphere (droplet). Something to keep in mind when determining what type of spatter, you are dealing with is that chords diameter is completely dependent on the velocity of the drop. When a blood drop falls it will accelerate according to the gravitational force. It will then continue to fall until reaching equilibrium with gravity and then come to a uniform velocity this is known as the Theory of Terminal Velocity. This theory was applied to blood spatter in order to gain some sort of reference sample of a constant velocity and known angles. However, the problem in using this theory is that it depends on the mass of the blood drop. Blood is not uniform because people have different proportions of blood composition. Due to these different Proportions variance is expected within the individual because of the different ratios within the many different organs.

2.2 DIFFERENT TYPES OF BLOOD SPATTER

2.2.1 Low Velocity Spatter

Low velocity spatter is one we are familiar with; it is anywhere from 1.5m/s or less. This Spatter is formed after we have “received” our wound. For example, if I was to get cut in the arm and blood began to drip from the wound, I Would then walk towards my first aid kit. The blood which hit the ground would have Simply been dripping from my arm; there was no force applied to cause it to do anything Else other than drip.

The drops which landed on the ground would be droplets. When the drop is travelling

Slowly the diameter of the chord will be wider. What is interesting about blood is that it will increase in size the greater the distance is at which it falls, but Will remains constant after 4ft. When dealing with low velocity spatter, we can apply the theory of terminal Velocity. Why? We can do this because terminal velocity does not deal with force; As you would have to consider when dealing with fast velocity blood spatter. This type of blood spatter gives an almost perfect sphere, but there are a few tails called Spines surrounding the bloodstain left behind. These stains are about 3mm or larger Diameter.

2.2.2 Medium Velocity Spatter

Medium velocity spatters are produced with more energy and force than low velocity spatter. This is because medium velocity deals with force, whereas low velocity is dependent on gravitational force. Medium velocity spatter is given by blunt force and stabbings.

When droplets are dispersed, they break off into smaller droplets of blood. When dealing with a stabbing, the bloodstain pattern will be relatively linear. This is because the surface area of the object is small, and less blood being deposited from the wound. However, when dealing with blunt force, the blood spatter left will be varied in size because the surface area is larger.

2.2.3 High Velocity Spatter

Forensics.com). The spatter which is associated with high velocity is 2mm in diameter or less, and the force that produces this spatter is

100ft/s! High velocity spatter deals with gunshots, high speed collision, and explosions.

The bloodstain pattern given off by high velocity spatter looks like a mist,

Because of the high velocity. Let us note that when we say velocity we mean the measure of force that has been applied to the blood. With some high velocity spatter some of the spatter may travel backward toward the gun, known as Back Spatter. However, if the spatter moved in the direction that the bullet was travelling this would be known as Forward Spatter. When the gun is closer to its target the spatter will be greater in dispersion, the same applies to the bullet, the larger it is the more dispersion there is. The spatter tends to be 2mm to 4mm in diameter, and the force which causes this type of spatter is 25ft/s (Crime scene).

2.3 ANGLE OF IMPACT:

In Forensic science, the angle of impact is the angle at Which a blood droplet impacts a surface. By Measuring the width and length of the stain, the angle of impact can be calculated, helping investigators determine the actions that may have taken place at the scene. Determination of the angle of impact of a blood stain Formula to measure an impact of a blood stain Angle of impact, $\theta = \sin^{-1}$ (width/length)

As the angle of impact is made smaller or more acute, the blood stain pattern will become more elongated, elliptical, or oval in shape.

2.3.1 To calculate the angle of impact

We take the width measurement and the length measurement of the blood stain, keeping in mind not to measure the tail. We do not measure the tail because it is caused by gravitational force and the force of the weapon. We should expect to see a larger tail the smaller the angle of impact. To calculate, do the following angle of impact tells us about the angle at which the blood hit the surface. To calculate do the following

Ex 1: Let us say that the width is 9mm and the length is 18mm STEP 1:

$sin-1$(9mm/18mm) = 30

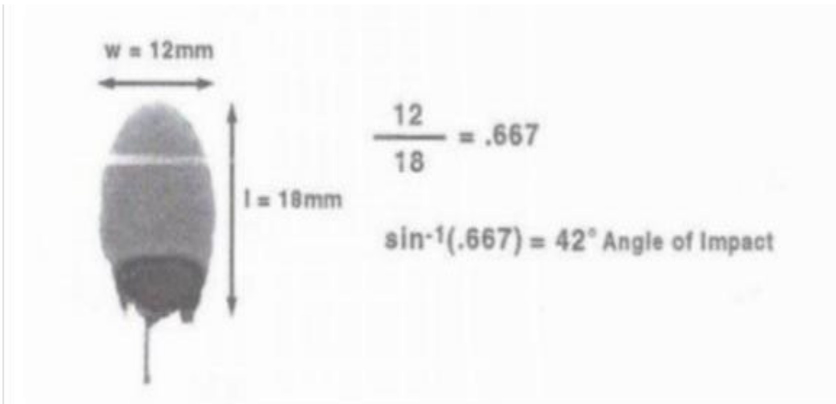
Ex. 2: Let us say the width is 1.5 cm and the length of the blood stain is 3.0cm

STEP 1: $sin-1$(1.5cm/3.0cm) = 30 The angle of impact was 30.

EX 3: Let us say that the width

Is12mm and Length is 18mm

Sine of impact angle=width of Bloodstain/length of bloodstain



2.4 AREA OF CONVERGENCE

The area of convergence tells us where the spatter may have originated. To do these strings are taken and are attached to each blood stain down its axis, this will show us where they Converge after stringing, we are able to see if the spatter is moving in an upward direction or a downward direction. Sometimes looking at the tail isn't enough because the gravitational force will simply pull it down anyway. When dealing with blunt force blood spatter, it is important to look up to the ceiling, as the blood spatter there is likely from the blood that was on the object and was put there due to the swinging of the object. It is important to look at Directionality, because this will show you which spatter are due to swinging the object backward and which is due to forward motion. If we know where the perpetrator was, we can get a much clearer Understanding on whether the person is right-handed or left-handed. There are times when the assailant will attempt to make a killing appear as a suicide. Area of convergence plays a very big role in this. It shows if there are any discrepancies. The beautiful thing about math is that it doesn't lie. If the area of convergence doesn't match where the body should be, this is a good time to start thinking that foul play may be involved. There are several times in which there is no blood spatter, but a gun was used for the "suicide". Because a gun is high velocity it is likely that the spatter is small.

2.5 Trigonometry

Trigonometry is the mathematical analysis of problem involving angles. Often using the trigonometric functions like sine, cosines and tangent. Within forensic science we may need to interpret data arising from measurements made at a crime scene.

For example, in blood pattern analysis or bullet ricochet or to investigate cases of forensic relevance such as the trajectory of a riffle bullet or a suspicious death resulting from a fall from a tall building in all these investigations we need to understand the basic principle of Trigonometry.

2.5.1 DETERMINING HEIGHT

It is important to determine the height at which the blood fell, because it tells us the Height that the blood drop originated from. It is possible that the victim and the assailant Both leave spatter evidence. Due to this, it is important to take notes of the heights and find where any inconsistency may lay. Knowing the height of the victim we may deduce t the angle of impact is 70 .° To determine the height we construct the Following.

Ex: Let's say that the blood stain was 15ft. away and the angle of impact was 80

STEP 1: tan 80 = X/15

STEP 2: 15(tan 80) = 85.1ft.°

The height at which the blood fell was 85.1ft.

Tangent of angle of impact = opposite / adjacent = height / distance

Height = tangent of angle x distance

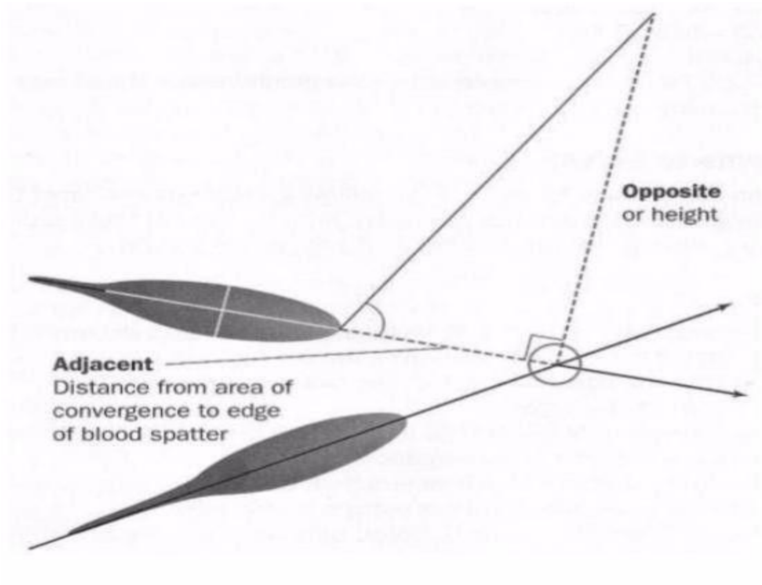
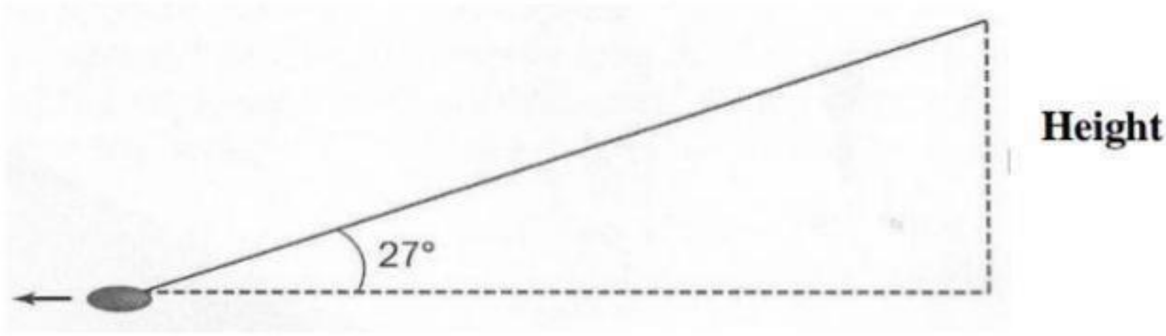


FIGURE B

Example:

Crime scene investigators noted blood spatter on the floor of the kitchen. The investigators drew lines of convergence and measured the distance from the area of convergence to the front edge of a drop of blood. That distance was recorded as 5.75 feet. After measuring the length and width of the blood droplet and using the law of sines, it was determined that the angle of impact was 27 degrees. The police wanted to determine the point of origin, or the height from the floor where the person was bleeding.



Distance to area of convergence = 5.75 ft

FIGURE C

Solution:

Tan = opposite / adjacent =height / distance Or Tangent of blood-spatter angle = height of wound / distance from blood to area of convergence tan 27*o* = height of wound / distance = height / 5.75 ft

Height = tan 27*o* x 5.75 ft Height

= 2.9 ft

Conclusion

As you have seen, Trigonometry has a large and important role in bloodstain analysis.

Without it we would not be able to find the height, angle of impact or area of convergence. Unfortunately, there are murders, and unfortunately for the murderers trying to flee there is trigonometry. Where there is blood stain, there is trigonometry and “absence of evidence is not evidence of absence.”

2.6 APPLICATION OF HEIGHTS AND DISTANCES IN CRIMINAL INVESTIGATION

Here we are investigating how direct application of Trigonometry may be used to solve some simple problem in forensic science .In determining heights and distances From a set of measurements the basic principle is that each example may be reduced to triangle or group of triangle and the Trigonometric functions and rules are then used to calculate any unknown length or angles Example:

A burglar gain access to an up-stair window by using a ladder. If the height of the window Is 4.3m above the ground and impression mark from the ladder are found 1.2m out from the wall. Calculate the length of the ladder and the angle it makes to the ground? The ladder forms the hypotenuse of a right-angled triangle with the ground and the wall. Let the length of the ladder be L, we use Pythagoras theorem to calculate

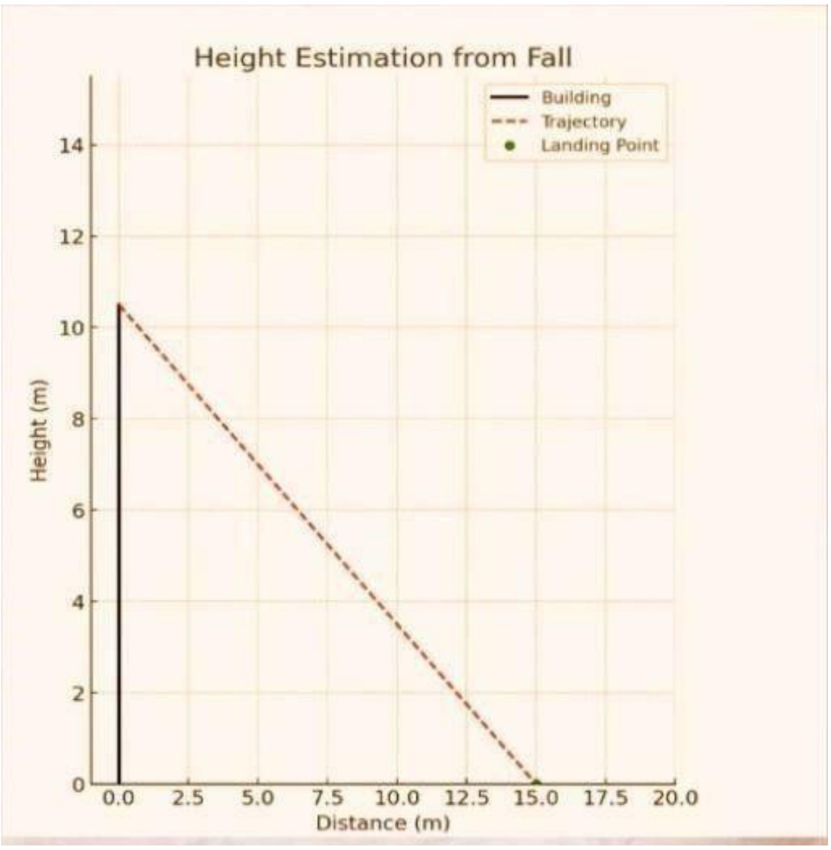
L L 2 =4.32 +1.22 = 19.93 L = = 4.46m

Since the angle in question is opposite the wall, the sine of this angle θ is given by

sin θ = 4.3 4.46 = 0.964 θ= sin−1 0.964 = 74.6

2.7 Crime scene reconstruction

One of the key uses of trigonometry in forensics is in crime scene reconstruction, particularly in analyzing trajectories and determining the positions of various elements involved in a crime. By examining the angles and distances of bullet paths, blood splatter, or the trajectory of objects, forensic experts can piece together the sequence of events. Suppose a forensic analyst is trying to determine the height from which a victim fell from a building. The analyst finds that the body landed 15 meters away from the base of the building. Assuming the angle of impact on the ground relative to the point directly below the point of impact is 35 degrees, calculate the height of the building. Figure illustrates the scenario where a forensic analyst is determining the height of a fall based on the landing distance and the angle of impact. The building is represented as a vertical line, and the trajectory of the fall is shown as a dashed red line. Ending at the landing point marked with a green circle. This visual representation, combined with trigonometric calculations, clearly demonstrates how the given distance from the base of the building (15 meters) and the angle of impact (35) degrees) are used to estimate the height of the building from which the victim fell. This problem can be solved using the tangent function in trigonometry, where $\tan(\theta) = opposite/adjacent$



Here, θ is the angle of impact, the opposite side is the height of the building, and the adjacent side is the distance from the base (15 meters). Therefore, the height from which the victim fell is approximately 10.5 meters.

2.8 Measurement of time of death

Time of death is a critical aspect of forensic science, playing a vital role in criminal investigations. It helps establish a timeline of events and can support or contradict witness statements and alibis.

Understanding the time of death is essential for forensic investigations, as it can significantly influence the outcome of a case. Accurate estimations require a combination of scientific techniques, experience, and careful analysis of all available evidence.

The main objectives of study are

1. To show how differential equations can be utilized to solve some physical problems

2. To investigate the application of differential equation in mathematical modelling3. To translate some physical situation in to mathematical problem and to solve the resulting problem.

2.8.1 The time of death

There are several times of death. It seems to be simple and straight forward term that obviously means the exact time that the victim draws his last breath. There are actually three different times of death.

1. Physiological time of death, where the victim’s vital functions actually cease.

2. The legal time of death, the time recorded in the death certificate.

3. The estimated time of death, the time medical examiner estimate that death has

   Occurred

If you are asked to tell the estimated time of death. We use the concept of Newton’s law of cooling

Newton’s law of cooling is differential equations that takes two inputs first the temperature at a given time and second the temperature at any arbitrary time. Because this equation can have infinitely many outcomes depending on the inputs, there is a constant which is associated with it. This constant can only be found from the two inputs and the equation can only be solved with the constant.

Newton's law of cooling relates the temperature of an ordinary body and the temperature of the body's surrounding medium, which is the source of the heat induction or heat loss of the body, and the change in temperature. The constant is the unique factor that depends on a particular solution.

2.8.2 Newton's law of cooling

The rate at which the temperature T(t) changes in a cooling body is directly proportional to the temperature difference between the body and the temperature TS of the surrounding medium.

Newton's law of cooling modelled as first order initial value problem

dT/dt=K(T-Ts)

$T(0)=T_0$

T_0 is the initial temperature of the body and K is the constant of proportionality If Ts is

Constant by method of separation of variables

$1/\ (T - Ts)\ Dt = K\ dt$

Integrating both sides

$\int 1/T-Ts = \int K\ dt$

Log $(T - Ts)$ = Kt + c

(T-Ts) = e^{Kt+c}

$T(t) = Ts + ce^{Kt}$

2.8.3 Postmortem Cooling

During life, body temperature is maintained by the body's metabolism. At death metabolism ceases and body temperature begins to approach the ambient temperature which is always lower than body temperature. Body warmth is lost from the surface to the surroundings, the core temperature remains relatively static for the first one to two hours after death
Normal body core temperature is 98.6 F and certainly many individuals die with that particular body temperature. However, there are factors may raise or lower a person's body temperature at death.
According to splitzetal body cooling is fairly complex and relays on a variety of body mechanisms. The factors that affect body temperature after death are humidity, insulation, surface in contact with the body, environment temperature
Careful consideration of all factors will not allow a pinpoint documentation of time of death but at least a range for time of death can be found. The range of death is the period of time in which the death is believed to have occurred.
When estimating the time of death, one need to know the temperature of the surroundings and the body temperature in order to make an accurate estimate

EXAMPLE 1

At 9am on October 19, 2009, a body was found in room 327 at the university centre. The room is kept at a constant temperature of 72F. The medical examiner was called, and he arrived in 8 minutes. The first thing he did was to take the temperature of the body, it was 83F. Thirty minutes later the temperature of the body was taken again, and it was now 78F. Help the police by telling them when the person was murdered?

The initial condition is taken as

T_0=83 and T_s=72 at 9.08 am

Substituting these values in

T(t) = Ts + (T_0 - Ts) e^{Kt} = 72 +(83-72) e^{Kt}

=72 +11$e^{Kt} \rightarrow eq$ 1

At 9.38 when the medical examiner took the body temperature T(30)=78 and Ts =72 again substitute as above we get

78= 72+ 11e^{Kt}
78- 72=11e^{Kt}

6=11 e^{Kt}

6 /11 = e^{Kt}

log 6 /11 = Kt

Thus, we get K=-0.0202 where t= 30

Substituting K in eq1

T(t)= 72+ 11e $-0.0202t$ for t

This equation takes the form of exponential decay due to its negative exponent.

Since the medical examiner wishes to know the time of death a solution for t is found by setting the temperature of body at 98.6F, normal body temperature

Now solving we get

98.6= 72+ 11$e^{-0.0202t}$ for t

98.6 – 72= 11$e^{-0.0202t}$ 26.6

/11 = $e^{-0.0202t}$

log 11/ 26.6 = 0.0202t t = log 11

26.6/ 0.0202 = - 43.7

Subtracting 43 minutes from 9.08 am gives the time of death.

That is 8.25 am

2.9 DNA Analysis-Fingerprinting

DNA analysis and fingerprinting are essential forensic tools used to identify individuals in criminal investigations. While DNA analysis primarily relies on biological methods, geometric principles play a crucial role in fingerprint analysis and pattern recognition.

Fingerprints exhibit unique patterns that can be categorized into three main types: whorls, loops, and arches.

According to H. M. W. Henneman et al. (2014), the classification of these patterns is essential for systematic identification and comparison in forensic science.

The geometric description of these patterns helps forensic analysts effectively categorize and compare fingerprints. Minutiae Points - Minutiae are the unique features found in fingerprints, including bifurcations (where a ridge splits) and ridge endings (where a ridge terminates). As noted by Lee and Gaensslen (2013), minutiae points are critical for establishing identity due to their distinctiveness. - Each fingerprint has a specific geometric arrangement of minutiae points, which forensic experts analyze to differentiate individuals. The spatial relationships between these minutiae points are vital for determining similarities among different fingerprints. Distance Measurements

Forensic scientists utilize geometric measurements to quantify the relationships among minutiae points. The research by Jain et al. (2006) emphasizes the importance of calculating the Euclidean distance between points and assessing the angles formed by groups of minutiae. This quantitative analysis aids in matching fingerprints against databases.

Techniques such as the minutiae matching algorithm leverage these geometric principles to enhance the accuracy of fingerprint identification.

Automated Fingerprint Identification Systems (AFIS)

AFIS utilizes geometric algorithms to automate the fingerprint matching process. As outlined by Maltoni et al. (2009), these systems convert fingerprint images into digital data, focusing on the geometric arrangement of minutiae points.

- When a fingerprint is submitted for comparison, AFIS analyzes the geometric features and spatial relationships to quickly identify potential matches, significantly improving the efficiency of investigations.

Conclusion

Incorporating geometric principles into DNA analysis and fingerprinting enhances the accuracy and efficiency of forensic investigations. Understanding the geometric relationships among fingerprint features allows forensic scientists to make reliable identifications, significantly contributing to the pursuit of justice.

CHAPTER 3

PROBABILITY IN FORENSIC SCIENCE

The term probability is given to a formal measure of the certainty that a particular event or outcome will occur. In each case of probability, the result is based on an unbiased outcome here every possible result is equally likely.

In forensic science empirical probabilities are particularly important and sample may be derived from the data on height, fingerprint class, blood group, allele frequencies in DNA

3.1 CALCULATING PROBABILITY :

The fundamental assumption of many probabilities calculation is that all outcomes are equally likely and either head or tail up. But which of these outcomes will occur on given occasion cannot be predicted.

$$\text{Probability} = \frac{number\ of\ selected\ outcomes}{total\ number\ of\ possible\ outcomes}$$

3.2 APPLICATION: PROBABILITY AND HUMAN TEETH MARKS:

The matching of bite marks, for example from indentations on the skin of a victim, against a reference set of marks from a suspect may be based on quantitative measurements of the position and orientation of each tooth with each set. For each tooth there are six classes of position, termed the bucal, lingual, mesial, distal, mesial rotation and distal rotation. However, detailed examination and measurement of 384 perfect sets of test bites has revealed that greater resolution was possible in terms of determining the position within the mark and the orientation angle of each tooth. These parameters were shown to follow frequency distributions with steep cut- off at the edges. By assessing the errors in such measurements as ±1 mm in determining the centre of the tooth and $\pm 5^0$ in each angular orientation, these distributions suggested that between

100 and 200 distinct positions were available for by each of the 12 teeth involved in this study. The average number of distinct positions was approximately 150. The total number of distinct positions over this set of 12 teeth is therefore given by 150^{12}

If it is assumed that the observed frequency distributions give some support to the proposition that each tooth position is equally likely, then the probability of any single distinct set of 12 tooth marks is given by:

$$P = \frac{1}{150^{12}} = 1.3 * 10^{-26}$$

As the global population is around 6700 million, such a probability justifies the conclusion that a well resolved set of bite marks may be individualized through this methodology

.

3.3 APPLICATION: THE MATCHING OF HAIR EVIDENCE

The macroscopic examination of hair evidence involves the individual assessment of many features of each hair such as color pigment, density, medulla characteristics and physical dimension. The forensic scientist will normally carry this as a comparison process between the pair of hair.

The work by GAUDETTE AND KEEPING has shown that for an experience scientist working on paired Caucasian, scalp hairs from different individual, nine pair were indistinguishable of all 366630 paired hair comparisons. This gives the probability that any two hairs taken at random from each of two people will be indistinguishable as

$$P = \frac{9}{366630} = 2.45 * 10^{-5}$$

This implies that the probability of distinguishing this hair is 1-P. It is common practice to compare a questioned hair with a set of n randomly chosen dissimilar reference hair from the same head using the first rule of combining probabilities. The probability that all n will be distinguished from the questioned hair is given by

$$P_n = (1 - P)^n = 1 - np$$

The approximation is valued when P≤ 1 and is called the binomial expansion approximation. This probability applies to a result where hairs from different individuals are correctly discriminated. Thus, the complimentary probability that the questioned hair is indistinguishable from at least one of the n reference hair is simply nP.

3.4 PROBABILITY IN SOLVING CRIMES:

According to Kim Rossmo, his research resulted in a new criminal investigative methodology called geographic profiling, based on Rossmo’s formula. His formula aims to find the location of where the offender might be living. Based on the studies, it is seen that criminals commit their crimes in an area not too close but also not too far away from where they live. We can find an area of possible locations:

In the diagram, the green spots highlight where the crimes have been done. The area called "HOT ZONE" is the area with the highest probability where the criminal resides.

He uses the following formula:

$$p_{i,j} = k \sum_{n=1}^{\text{(total crimes)}} \left[\underbrace{\frac{\phi_{ij}}{(|X_i - x_n| + |Y_j - y_n|)^f}}_{1^{\text{st}}\ \text{term}} + \underbrace{\frac{(1-\phi_{ij})(B^{g-f})}{(2B - |X_i - x_n| - |Y_j - y_n|)^g}}_{2^{\text{nd}}\ \text{term}} \right]$$

We divide the area into grids and the equation shows the probability in a particular grid.

The first term essentially the denominator, measures the distance from the point of crime till the grid.

HIGHER THE DISTANCE → HIGHER THE DENOMINATOR → HIGHER THE

PROBABILITY OF THE GRID BEING THE RESIDENCE AREA OF THE CRIMINAL.

The second term essentially the denominator, subtracts the distance from a buffer zone (B).

All of the other variables in the equation remain constant.

HIGHER THE DISTANCE → LOWER THE DENOMINATOR → SMALLER THE

PROBABILITY OF THE GRID BEING THE RESIDENCE AREA OF THE CRIMINAL.

Both these terms help balance the distance from being too close or too far from the crime scene. When we add these, we can make the HOT ZONE.

CHAPTER 4

CASE STUDY

4.1 Case study:

A real case study where calculus (specifically Newton's Law of Cooling) was applied to estimate the time of death is relatively rare due to the complexities of crime scene investigations and the reliance on multiple forensic methods. However, there are notable instances where temperature data, in combination with calculus, has been used to estimate the post-mortem interval (PMI). Below is a realistic case study based on an actual forensic approach where calculus was employed for determining the time of death:

4.2 Case Overview:

Realistic Case Study: The Death of John Doe

Background:

A 40-year-old male, referred to as "John Doe," was found deceased in his apartment. The body was discovered by a family member who had become concerned after not hearing from him for a few days. The body was not in an advanced state of decomposition but had been dead for a number of hours. The investigation was part of a homicide inquiry, as there was no obvious sign of natural causes. The police needed to estimate the time of death to determine the timeline of events and potentially find witnesses or suspects.

Scene Details:

Date of discovery: June 12, 2023, 9:00 AM

Ambient temperature (environmental temperature): 22°C (71.6°F)

Body temperature at discovery: 26°C (78.8°F)

Normal body temperature at death: 37°C (98.6°F)

Assumed rate constant (k): $k = 0.02\ hr^{-1}$ (this value was based on empirical studies of the environment and body composition)

Step 1: Use Newton's Law of Cooling

We use Newton's Law of Cooling which describes how the temperature of an object (in this case, the body) changes over time. The equation is:

$T(t) = T_{ambient} + (T_0 - T_{ambient}\ e^{-kt}$

Where:

$T(t)$ is the temperature of the body at time t ,

$T_{ambient} = 22°$ C (ambient temperature),

$T_0 = 37°$ C (initial body temperature at the time of death),

$k = 0.02hr^{-1}$(rate constant),

t is the time in hours since death, which we need to determine.

Step 2: Set up the Equation for Time of Death

Given:

$T(t) = 26°C$ (the measured temperature at the time of discovery),

• $T_{ambient} = 22°$ C ,

• $T_0 = 37°$ C ,

• $k = 0.02\ hr^{-1}$.

We substitute these values into the formula:

$$26 = 22 + (37 - 22)\ e^{-0.02t}$$

Simplifying:

$$26 - 22 = 15\ e^{-0.02t}$$

$4 = 15\ e^{-0.02t}$

Now, solve for t :

$e^{-0.02t} = 4/15$

Taking the natural logarithm of both sides:

$-0.02t = \ln 4/15$

$-0.02t = \ln(0.2667)$

$-0.02t = -1.3217$

Now solve for t :

$T = 1.3217/0.02 = 66.085$ hours

Step 3: Interpretation of Results

The time of death is estimated to be approximately 66 hours before the body was discovered, which corresponds to around 3 days before the discovery.

Real-World Implications and Assumptions:

Rate constant k : The constant $k = 0.02\ hr^{-1}$ is an estimate based on empirical studies. This value can vary depending on factors such as the environment (indoor vs. outdoor), clothing, air circulation, and the body's composition. In real cases, forensic investigators often need to adjust this value based on environmental conditions, or they use tables and software tools that factor in these conditions.

Assumptions: This calculation assumes that the temperature cooling process follows Newton's Law precisely. In real-world cases, deviations can occur due to external factors such as humidity, wind, and the victim's health prior to death.

Other Methods Used in the Case: In addition to the temperature-based estimate, investigators would have likely considered other forensic method like rigor mortis, livor mortis, and entomological evidence to provide a range of possible times of death.

Conclusion:

In this case, using calculus and Newton's Law of Cooling, the time of death was estimated to be 66 hours before the body was discovered. This method provided an initial estimate that would guide investigators in forming a timeline of events and potentially identifying suspects or witnesses who may have interacted with the victim within that time frame.

Limitations and Challenges:

While Newton’s Law of Cooling provides a useful model for estimating the time of death, it is not always precise, especially in complex real-world situations where environmental factors are unpredictable. In practice, the method would be corroborated with other evidence such as witness testimonies, forensic pathology reports, and entomological data to give a more accurate estimate.

Reference

❖ **Essential mathematics and statistics for forensic science; CRIAG ADAM**

❖ **The determination of the estimated time of death using differential equation; FAITHU MUHAMMED AND AUWALU HARUNA**

❖ ❖**wikipedia**