TB214090W

Reg. No :...............................

Name :.................................

B.Sc. DEGREE (C.B.C.S.) EXAMINATION, MARCH 2023

(2021 Admissions Regular, 2020 Admissions Supplementary / Improvement, 2019 &2018 Admissions Supplementary)

SEMESTER IV - COMPLEMENTARY COURSE 1 (MATHEMATICS)

(For Physics & Chemistry)

MT4C01B18 - FOURIER SERIES , PARTIAL DIFFERENTIAL EQUATIONS, NUMERICAL ANALYSIS AND ABSTRACT ALGEBRA

Time : 3 Hours **Maximum Marks : 80**

Part A

I. Answer any Ten questions. Each question carries 2 marks **(10x2=20)**

1. Write the equation to find half range Fourier Cosine series.
2. Write the Rodrigues's Formula and show that $P_1(x) = x$.
3. Show that $P_3(x) = \frac{1}{2}(5x^3 - 3x)$
4. Write the equation to find the tangent plane at a point P (x,y,z) to the surface S whose equation is F(x,y,z)=0.
5. Eliminate the arbitrary function f from the equation $z = x + y + f(xy)$.
6. Write the Langrange's subsidiary equations for $x^2\frac{\partial z}{\partial x} + y^2\frac{\partial z}{\partial x} = (x+y)z$.
7. Define an algebraic equation. Give an example.
8. Check whether there exists a root between 2 and 4 for the equation $2x^2 - 5x + 8 = 0$
9. Define Group Homomorphism
10. If a and b are any two elements of a group G then show that $(a*b)^{-1} = b^{-1}*a^{-1}$
11. Compute r^2 where $r = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 3 & 5 & 4 & 2 & 1 & 6 \end{pmatrix}$
12. Find the inverse of the permutation $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 5 & 3 & 4 & 6 & 1 & 2 \end{pmatrix}$

Part B

II. Answer any Six questions. Each question carries 5 marks **(6x5=30)**

13. Find the Fourier series of f given by $f(x) = x, -\pi < x < \pi$ and $f(x) = f(x + 2\pi)\ \forall x \in R$.
14. Derive: $J_0'(x) = -J_1(x)$
15. Solve the differential equation $9x^2y'' - 27xy' + (9x^2 + 35)y = 0.$
16. Show that $J_2'(x) = \frac{1}{2}[J_1(x) - J_3(x)]$.

17. Find the integral curves of the equations $\frac{dx}{z} = \frac{dy}{-z} = \frac{dz}{z^2 + (x+y)^2}$.

18. Apply Newton Raphson method to solve the algebraic equation $x^3 - 5x + 3 = 0$ correct to three decimal places.

19. Prove that intersection of two subgroups is again a subgroup of G

20. Define a ring. Check whether $< \mathbb{Z}, +, . >$ is a ring where '+' is the usual addition, ' . ' is the usual multiplication and Z is the set of integers.

21. If $\mu = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 5 & 2 & 4 & 3 & 1 & 6 \end{pmatrix}$. Compute μ^{100}

Part C

III. Answer any Two questions. Each question carries 15 marks **(2x15=30)**

22. Find the two half range expansions of $f(x) = \begin{cases} \frac{2kx}{L}, & 0 < x < \frac{L}{2} \\ \frac{2K}{L}(L-x), & \frac{L}{2} < x < L \end{cases}$.

23. (a) Find the integral curves of $\frac{dx}{x^2(y^3 - z^3)} = \frac{dy}{y^2(z^3 - x^3)} = \frac{dz}{z^2(x^3 - y^3)}$

    (b) Eliminate the arbitrary function f from the eqution $z = xf(2x - y) + g(2x - y)$.

24. Find a real root of the equation $x^3 - 2x - 5 = 0$ using the method of false position.

25. (a). Define $*$ on Q^+ by $a * b = \frac{ab}{2}$. Show that Q^+ under the operation $*$ is an abelian group.

    (b). Show that the set G ={1,-1, i, -i} forms an abelian group with respect to multiplication.