TB214180W

Reg. No :...............................

Name :.................................

B.Sc. DEGREE (C.B.C.S.) EXAMINATION, MARCH 2023

(2021 Admissions Regular, 2020 Admissions Supplementary / Improvement, 2019 &2018 Admissions Supplementary)

SEMESTER IV - CORE COURSE (MATHEMATICS)

MT4B04B18 - VECTOR CALCULUS, THEORY OF EQUATIONS & MATRICES

(For Maths & Comp. Applications)

Time : 3 Hours **Maximum Marks : 80**

Part A

I. Answer any Ten questions. Each question carries 2 marks (10x2=20)

1. Define unit tangent vector and unit normal vector. Write the formula to find each.
2. Find the velocity vector of $\bar{r}(t) = (t^2)\bar{\imath} + (-7t)\bar{\jmath} + (-1)\bar{k}$.
3. Find the vector parallel to the line of intersection of the planes $3x-6y-2z=15$ and $2x+y-2z=5$.
4. State Divergence theorem.
5. Find a parametrization of the sphere $x^2+y^2+z^2=a^2$
6. Determine whether $\bar{\mathbf{F}} = (2x-3)\bar{i} - z\bar{j} + (\cos z)\bar{k}$ is conservative.
7. State Division Algorithm for polynomial functions.
8. If α, β, δ are the roots of the equation $x^3-px^2+qx-r=0$. Find the value of $\frac{1}{\alpha^2}+\frac{1}{\beta^2}+\frac{1}{\gamma^2}$
9. If α, β, γ are the roots of $2x^3+x^2-2x-1=0$ then find the value of $\alpha\beta+\beta\gamma+\alpha\gamma$ and $\alpha+\beta+\gamma$
10. Define the Normal form of a matrix
11. Find the rank of the matrix $\begin{bmatrix}1&2&3\\2&3&4\\3&5&7\end{bmatrix}$.
12. Write the characteristic equation of the matrix $\begin{bmatrix}2&2&0\\2&2&0\\0&0&1\end{bmatrix}$

Part B

II. Answer any Six questions. Each question carries 5 marks (6x5=30)

13. The acceleration of a particle at time t is given by $\vec{a} = 12\cos 2t\,\vec{i} - 8\sin 2t\,\vec{j} + 16t\,\vec{k}$, Find velocity $\vec{v}$ and displacement $\vec{r}$ at time t if the velocity and displacement is zero at time t=0.
14. Find the equation for the tangent to the ellipse $\frac{x^2}{4}+y^2=2$ at the point (-2, 1).
15. State the tangential and normal forms of Green's theorem.
16. Find a potential function for the field $\bar{\mathbf{F}} = 2x\bar{i} + 3y\bar{j} + 4z\bar{k}$
17. Show that $\bar{F} = (e^x\cos y + yz)\bar{i} + (xz - e^x\sin y)\bar{j} + (xy+z)\bar{k}$ is conservative and find a potential function for it.
18. Prove that every polynomial equation of nth degree has exactly n roots
19. Solve $x^4+2x^3-21x^2-22x+40=0$ given that the sum of two if its roots is equal to the sum of the other two

20. Find the eigenvalues and corresponding eigenvectors of the matrix $A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$.

21. Find the rank of the matrix $A = \begin{bmatrix} 1 & 6 & -18 \\ -4 & 0 & 5 \\ -3 & 6 & -13 \end{bmatrix}$ by reducing it to the Echelon form.

Part C

III. Answer any Two questions. Each question carries 15 marks **(2x15=30)**

22. Find T, N, B, κ, τ for the space curve $r(t) = e^t \cos t\, \vec{i} + e^t \sin t\, \vec{j} + 2\vec{k}$.

23. Find the area of the surface cut from the bottom of the paraboloid $x^2 + y^2 - z = 0$ by the plane $z = 4$

24. Solve $6x^5+11x^4-33x^3-33x^2+11x+6 = 0$

25.

a) Determine the eigenvectors of the matrix $A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$

b) If $A = \begin{bmatrix} 1 & 1 & 2 \\ 3 & 1 & 1 \\ 2 & 3 & 1 \end{bmatrix}$, find A^{-1} and A^3.