TB222350W

Reg. No :...............................

Name :.................................

B. Sc. DEGREE (C.B.C.S.) EXAMINATION, MARCH 2023

2022 Admissions Regular & 2021 Admissions Supplementary / Improvement And 2020, 2019 And 2018 Admissions Supplementary

SEMESTER II - COMPLEMENTARY COURSE 1 (MATHEMATICS)

(For PHYSICS and CHEMISTRY)

MT2C01B18 - PARTIAL DERIVATIVES, MULTIPLE INTEGRALS TRIGONOMETRY AND MATRICES

Time : 3 Hours **Maximum Marks : 80**

Part A

I. Answer any Ten questions. Each question carries 2 marks **(10x2=20)**

1. Solve $\int_0^2 \int_0^2 \int_0^2 xyz\ dxdydz$
2. Change the Cartesian integral $\int_0^6 \int_0^y x\ dxdy$.
3. Change to polar coordinates: $\int_{-1}^1 \int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} ln(x^2+y^2+1)\ dxdy$.
4. Prove that $cos(i\theta) = cosh(\theta)$
5. If $z = e^{i\alpha}$, show that $\frac{z^2-1}{z^2+1} = i\ tan\alpha$.
6. Show that $cosh\ 2x = 2cosh^2\ x - 1.$
7. Write the chain rule of differentiation for two independent variables and three intermediate variables.
8. Find $\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}$ and $\frac{\partial f}{\partial z}$ if $f(x,y,z) = 1 + xy^2 - 2z^2$.
9. Find $\frac{\partial f}{\partial y}$ if $f(x,y) = y\ sin\ xy$.
10. Differentiate between consistent and inconsistent systems.
11. Find the eigen values of the matrix $\begin{bmatrix} 3 & 4 \\ 5 & 2 \end{bmatrix}$
12. FInd the characteristic equation of the matrix $\begin{bmatrix} 1 & 1 & 2 \\ 3 & 1 & 1 \\ 2 & 3 & 1 \end{bmatrix}$.

Part B

II. Answer any Six questions. Each question carries 5 marks **(6x5=30)**

13. Find the area of the region enclosed by the line $y = 2x + 4$ and the parabola $y = 4 - x^2$ in the $xy - plane$.

14. Evaluate the area enclosed by the lemniscate $r^2 = 4\ cos2\theta$.

15. If $(x + iy) = cosh\ (u + iv)$, show that

$$(i) \frac{x^2}{cosh^2 u} + \frac{y^2}{sinh^2 u} = 1$$
$$(ii) \frac{x^2}{cos^2 v} - \frac{y^2}{sin^2 v} = 1.$$

16. If $sin\ (\theta + i\ \phi) = tan\ (x + iy)$, show that

$$\frac{tan\ \theta}{tanh\ \phi} = \frac{sin\ 2x}{sinh\ 2y}.$$

17. Verify that $w_{xy} = w_{yx}$, where $w = x\ siny + y\ sinx + xy$.

18. Find $\frac{\partial z}{\partial u}, \frac{\partial z}{\partial v}$ at $u = 0, v = 0$ if $w = x^2 + \frac{y}{x}$ and $x = u - 2v + 1,\ y = 2u + v - 2.$

19. Find the rank of the matrix by reducing to its normal form

$$A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 3 & 4 \\ 4 & 3 & 2 & 1 \end{bmatrix}$$

20. Check whether the system of equations are consistent or not :

$$x - 4y + 7z = 14$$
$$3x + 8y - 2z + 13$$
$$7x - 8y + 26z = 5.$$

21. Find the eigen values and corresponding eigen vectors of the matrix $A = \begin{bmatrix} 7 & -2 & 2 \\ -2 & 1 & 4 \\ -2 & 4 & 1 \end{bmatrix}$

Part C

III. Answer any Two questions. Each question carries 15 marks **(2x15=30)**

22. (a) Find the volume of the region D enclosed by the surfaces $z = x^2 + 3y^2$ and $z = 8 - x^2 - y^2$.

(b) Evaluate $\int_0^\pi \int_0^\pi \int_0^\pi cos(u + v + w)\ du\ dv\ dw\ (uvw - space).$

23. (a) Derive the expansion of $tan(n\theta)$.

(b)Sum the series : $cos\ \alpha +^n C_1 cos\ (\alpha + \beta) +^n C_2\ cos(\alpha + 2\beta) + ... + cos\ (\alpha + n\beta)$.

24. (a) Find $\frac{\partial w}{\partial r}, \frac{\partial w}{\partial s}$ when $r = 1,\ s = -1$ if

$w = (x + y + z)^2,\ x = r - s,\ y = cos(r + s),\ z = sin(r + s).$

(b) Find $\frac{dy}{dx}$ of the function $f(x, y) = x^2 + xy + y^2 - 7 = 0$ at the point $(1, 2)$ using implicit differentiation.

(c) Show that the function $f(x, y, z) = 2z^3 - 3(x^2 + y^2)z$ satisfies the Laplace Equation $\frac{\partial f}{\partial x^2} + \frac{\partial f}{\partial y^2} + \frac{\partial f}{\partial z^2} = 0$.

25. Solve by Cramer's rule :

$$
\begin{aligned}
2a + b + 5c + d &= 5 \\
a + b - 3c - 4d &= -1 \\
3a + 6b - 2c + d &= 8 \\
2a + 2b + 2c - 3d &= 2
\end{aligned}
$$