TB222360W **Reg. No :.....**

Name :.....

B. Sc. DEGREE (C.B.C.S.) EXAMINATION, MARCH 2023

2022 Admissions Regular & 2021 Admissions Supplementary / Improvement And 2020, 2019 And 2018 Admissions Supplementary

SEMESTER II - COMPLEMENTARY COURSE 1(MATHEMATICS) (For ECONOMICS)

MT2C02B18 - EXPONENTIAL, LOGARITHMIC FUNCTIONS, LINEAR ALGEBRA AND ADVANCED CALCULUS

Time : 3 Hours **Maximum Marks : 80**

Part A

I. Answer any Ten questions. Each question carries 2 marks **(10x2=20)**

1. Convert the natural logarithm $ln24 = 3.17805$ into natural exponential function.
2. Change the exponential function $12 = 144^{\frac{1}{2}}$ to logarithmic form.
3. Use the properties of logarithms to write the expression as sums or differences or products. $\frac{8\sqrt[4]{x}}{\sqrt{y}}$.
4. Find the dimension of the transpose of the matrix $\begin{bmatrix} 4 & 8 \\ 7 & 2 \\ 3 & 5 \end{bmatrix}$.
5. Find the determinant of the matrix $A = \begin{bmatrix} 8 & 3 \\ 9 & 2 \end{bmatrix}$.
6. Find the difference $A - B$, where $A = \begin{bmatrix} 6 & 3 & 7 \\ 2 & 9 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 5 & 8 & 9 \\ 4 & 1 & 2 \end{bmatrix}$.
7. Describe the Feasible region of a Linear programming problem.
8. Identify the objective function, technical constraints, non-negativity constraint and decision variables from the linear programming problem Minimize c = 50x + 30y subject to the constraints 2x+y <14; 5x+5y < 40; x+3y < 18; x and y are nonnegative.
9. Consider the linear programming problem Maximize f = 6x + 5y subject to the constraint 7x + y > 21 where x and y are nonnegative. Find a basic feasible solution for this LPP ?
10. Compute $\frac{dy}{dx}$ of the implicit function $6x^2 - 5xy + 32y = 2x$ using implicit function rule.
11. Evaluate $\frac{\partial z}{\partial a}$ at (0, 2) for the function $z = 2a^3b^6$.
12. For the function $z = \frac{6xy}{e^{5x+2}}$, compute $\frac{\partial z}{\partial x}$.

Part B

II. Answer any Six questions. Each question carries 5 marks **(6x5=30)**

13. A country's population P goes from 54 million in 1987 to 63.9 million in 1993. at what rate r is the population growing?
14. Differentiate :
    $(i) y = (ln\ x^2 + 7x + 15)$
    $(ii) y = ln^2\ 4x^3$

15. (a) Given tthe details for a company with several different retail stores selling the same line of products. Form a matrix using the given data.

Store	PC's	Printers	Monitors	Modems
1	120	145	130	85
2	165	105	155	90
3	110	115	95	80
4	185	170	165	105

(b) Given $D = \begin{bmatrix} 20 & 15 & 35 & 10 \\ 25 & 5 & 30 & 15 \\ 10 & 40 & 25 & 20 \\ 15 & 20 & 35 & 10 \end{bmatrix}$, where D represents deliveries made to the different stores. Find the new level of stock.

16. Given $A = \begin{bmatrix} 9 & 15 & 4 \\ 2 & -8 & 7 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & -8 & 13 \\ 6 & 11 & -5 \end{bmatrix}$.
    (a)Find the sum A+B
    (b) Find the difference A-B
    (c) Find the transpose of A and B.
17. An airline offers coach and first-class tickets. For the airline to be profitable, it must sell a minimum of 25 first-class tickets and a minimum of 40 coach tickets. The company makes a profit of $225 for each coach ticket and $200 for each first-class ticket. At most, the plane has a capacity of 150 travelers. Reduce this data to equations and inequalities suitable for maximizing the profit.
18. A bakery makes $4 profit on its wedding cakes A and $3 on its birthday cakes B. Wedding cakes take 4 minutes for mixing, 90 minutes for baking, and 8 minutes for icing. Birthday cakes take 6 minutes for mixing, 15 minutes for baking, and 4 minutes for icing. The bakery has 120 minutes of mixing time, 900 minutes of baking time, and 96 minutes of icing time. Express the data in terms of equations and inequalities necessary to determine the combination of wedding cakes and birthday cakes that will maximize profit subject to the constraints.
19. State the inverse function rule and illustrate it with the help of an example.
20. Compute the critical values at which the function $z = 6x^2 - 108x + 4xy + 12y - 2y^2 - 19$ is optimized and test the second-order conditions to see if the function is at a relative maximum, relative minimum, inflection point, or saddle point.
21. For the quadratic function $z = -7x^2 + 88x - 6xy + 42y - 2y^2 + 4$, find the critical points and determine whether at these points the function is at a relative maximum, relative minimum, inflection point, or saddle point.

Part C

III. Answer any Two questions. Each question carries 15 marks **(2x15=30)**

22. (a) Differentiate $y = ln(8x^2 + 3)$
    (b) Given a nomianl rate of interest $r = 8$ percent find the effective rate of interest under (i) semiannual (ii)quarterly, (iii)continous compunding, given the principal amount $P = Rs.3000$.

(c) Solve $log_a 32 = \frac{5}{3}$

23. (a) Use Cramer's rule to solve

$$-5p_1 + p_2 + 2p_3 = -6$$
$$2p_1 - 9p_2 + p_3 = -14$$
$$3p_1 + p_2 - 8p_3 = -58$$

(b) Using inverse Matrix method solve

$$8x_1 + 3x_2 = 14$$
$$9x_1 + 2x_2 = 24$$

24. Solve graphically the following linear programming problem:

Maximize :

$$\pi = 7x_1 + 21x_2$$

subject to the constraints :

$5x_1 + 2x_2 \leq 40; \quad 3x_1 + 12x_2 \leq 60; \quad 4x_1 + 8x_2 \leq 48; \quad x_1, x_2 \geq 0$

25. Optimize the function $z = 14x^2 - 9xy + 16y^2 + 835$ subject to the constraint: 3x + y = 148