B. Sc. DEGREE (C.B.C.S.) EXAMINATION, MARCH 2023
2022 Admissions Regular & 2021 Admissions Supplementary / Improvement And 2020, 2019 And 2018 Admissions Supplementary
SEMESTER II - CORE COURSE (MATHEMATICS)
(COMMON FOR MATHEMATICS AND COMPUTER APPLICATIONS)
MT2B02B18 - NUMBER THEORY, CRYPTOGRAPHY, LAPLACE TRANSFORMS & CONIC SECTIONS

Time : 3 Hours   **Part A**   Maximum Marks : 80

I. Answer any Ten questions. Each question carries 2 marks   **(10x2=20)**

1. If 3 doesnot divide n, then show that $\phi(3n) = 2\phi(n)$

2. Determine the remainder when 15! is divided by 17, using Wilson's theorem

3. Examine whether 1105 is an absolute pseudoprime

4. Define (a). Enciphering (b). Deciphering

5. Encipher the message "Happy days are here" using the autokey cipher with seed Q.

6. Determine whether the sequence 7, 27, 47, 97, 197, 397 is superincreasing

7. Compute $\mathcal{L}\{e^{-kt}(a\cos t + b\sin t)\}$

8. Find $\mathcal{L}\left\{\dfrac{1}{\sqrt{t}}\right\}$.

9. Find $\mathcal{L}\{e^{2t}\cos ht\}$

10. Write the polar equation for the circle $x^2 + (y+7)^2 = 49$.

11. Find the asymptotes of the hyperbola $y^2 - 3x^2 = 3$.

12. Find the focus and directrix of the parabola $x = 2y^2$.

Part B

II. Answer any Six questions. Each question carries 5 marks   **(6x5=30)**

13. If $a \equiv b \pmod{n}$ and $c \equiv d \pmod{n}$, then show that $(a+c) \equiv (b+d) \pmod{n}$ and $ac \equiv bd \pmod{n}$.

14. Show that $18! \equiv -1 \pmod{437}$

15. Explain Autokey cryptosystem. Give an example.

16. A user of a Knapsack cryptosystem has the sequence 49, 32, 30 43 as listed encryption key. If the user's private key involves the modulus m=50 and multiplier a =33. Determine the secret superincreasing sequence.

17. State and prove Convolution theorem for Laplace Transforms.

18. Compute $\mathcal{L}^{-1}\{F(s)\}$ where $F(s) = \dfrac{1}{(s+a)(s+b)}$

19. Calculate the inverse Laplace Transform of $F(s) = \dfrac{10}{s^3 - \pi s^2}$ using integration.

20. Find a Cartesian equation for the hyperbola having focus at $(0, \pm\sqrt{2})$ and the equation of the asymptotes $y = \pm x$.

21. Write the polar equation for the circle whose Cartesian equation is given by $(x-6)^2 + y^2 = 36$. Also sketch the circle.

Part C

22. (a). State Chinese Remainder Theorem.

    (b). Solve the linear congruence $17x \equiv 9 (mod\ 276)$ using Chinese Remainder Theorem.

23. (a). Write a short note on Hill's Cipher.

    (b). Use Hill's Cipher $C_1 \equiv 5P_1 + 2P_2 (mod\ 26)$, $C_2 \equiv 3P_1 + 4P_2 (mod\ 26)$ to encipher the message "GIVE THEM TIME".

24.
    (a). Solve the Volterra integral equation of the second kind $y(t) + \int_0^t y(\tau) \cos h(t-\tau) d\tau = t + e^t$.

    (b). Find the inverse Laplace Transform of $H(s) = \dfrac{1}{(s^2 + w^2)^2}$ by convolution.

25. (a). A wheel of radius a rolls along a horizontal straight line. Find the parametric equations for the path traced by a point P on the wheel's circumference. (b). Find a Cartesian equation for the hyprbola centered at the origin that has focus at (3, 0) and the line x=1 as the corresponding directrix.