TB206505W

Reg. No :...............................

Name :..................................

B. Sc. DEGREE (C.B.C.S.) EXAMINATION, MARCH 2023

(2020 Admission Regular, 2019, 2018 Admissions Supplementary)

SEMESTER VI - CORE COURSE (MATHEMATICS)

MT6B11B18 - GRAPH THEORY & FUZZY MATHEMATICS

Time : 3 Hours **Maximum Marks : 80**

Part A

I. Answer any Ten questions. Each question carries 2 marks **(10x2=20)**

1. Find the smallest integer n such that the complete graph K_n has atleast 500 edges.
2. What will be the diagonal entries of the adjacency matrix of a simple graph?
3. Prove that for any graph G there is an even number of odd vertices.
4. Show that star graphs are the only complete bipartite graphs which are trees.
5. Define a tree. Draw a tree with 3 vertices.
6. If G has 17 edges, what is the maximum possible number of vertices in G?
7. Calculate the number of different spanning trees of K_5.
8. Establish that the law of excluded middle is violated for fuzzy sets.
9. Define a fuzzy set. Give an example.
10. State the first decomposition theorem of fuzzy sets
11. State the characterization theorem of t - Conorms.
12. Prove that, if dual point of an element p in [0, 1] with respect to the fuzzy complement c is equal to c(p), then c is involutive.

Part B

II. Answer any Six questions. Each question carries 5 marks **(6x5=30)**

13. Given any two vertices u and v of a graph G, prove that every u - v walk contains a u - v path.
14. Draw the graph having the following adjacency matrix $\begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$.
15. Explain Chinese Postman Problem.
16. Let G be a graph with n vertices and q edges and let $\omega(G)$ denote the number of connected components of G. then show that G has at least $n - \omega(G)$ edges.i.e., $q \geq n - \omega(G)$.
17. Let e be an edge of a connected graph G, then prove that e is a loop if and only if it is in no spanning tree of G.
18. Prove or disprove: The standard fuzzy Union is a strong cut-worthy property when applied to a finite family of fuzzy sets.
19. Prove or disprove: The standard fuzzy intersection is a strong cut-worthy property when applied to an arbitrary family of fuzzy sets.
20. Prove that the function $i : [0, 1] X [0, 1] \rightarrow [0, 1]$ defined by i(a, b) = ab is a t-norm.
21. Prove that the function $u : [0, 1] X [0, 1] \rightarrow [0, 1]$ defined by u(a, b) = min {1, a + b } is a t-conorm.

Part C

III. Answer any Two questions. Each question carries 15 marks **(2x15=30)**

22. Let G be a non empty graph with at least two vertices. Then G is bipartite if and only if it has no odd cycles.
23. Let G be a graph with n vertices. Then show that the following three statements are equivalent. a)G is a tree. b) G is an acyclic graph with n-1 edges. c) G is a connected graph with n-1 edges.
24. State and prove the characterization theorem of convex fuzzy sets defined on the set of real numbers
25. Given a t-conorm *u* and an involutive fuzzy complement *c, then* prove that the binary operation on $[0,1]$ defined by $i(a,b) = c\left(u\left(c(a), c(b)\right)\right) \quad a, b \in [0,1],$ is a t-norm such that < i, u, c > is a dual triple.