TB206520W

Reg. No :...............................

Name :.................................

B. Sc. DEGREE (C.B.C.S.) EXAMINATION, MARCH 2023

(2020 Admission Regular, 2019, 2018 Admissions Supplementary)

SEMESTER VI - CORE COURSE (MATHEMATICS)

MT6B12B18 - LINEAR ALGEBRA

Time : 3 Hours

Maximum Marks : 80

Part A

I. Answer any Ten questions. Each question carries 2 marks **(10x2=20)**

1. Determine whether the set of three dimensional row matrices with all components real and equal is a vector space under regular addition and scalar multiplication if the scalars are complex numbers.
2. Write down the standard basis of $\mathbb{M}_{2\times 2}$.
3. Find the eigen values of $A = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 4 & -1 \end{bmatrix}$
4. Define a linear transformation $T : \mathbb{R}^2 \mapsto \mathbb{R}^2$ by $T\begin{bmatrix} a & b \end{bmatrix} = \begin{bmatrix} 2a & -b \end{bmatrix}$. Find $T\begin{bmatrix} 0 & 5 \end{bmatrix}$
5. Determine whether the linear transformation $T : V \mapsto W$ defined by $T(\bar{v}) = \bar{0}$ for all vectors $\bar{v}$ in V is linear.
6. Determine whether $\lambda_1 = 12$ and $\lambda_2 = -4$ are eigen values of $A = \begin{bmatrix} 11 & 3 \\ -5 & -5 \end{bmatrix}$
7. Define modal matrix and spectral matrix.
8. Two eigen values of a 3x3 matrix are known to be 5 and 8. What can be said about the third eigen value if the determinant of the matrix is -4 ?
9. Two eigen values of a 3 x 3 matrix are known to be 5 and 8. Predict the third eigen value, if the trace of the matrix is -8 .
10. Write down the formula to find the angle between two vectors.
11. Find the Euclidean inner product of $\bar{x} = \begin{bmatrix} -5 & 7 \end{bmatrix}^T$ and $\bar{y} = \begin{bmatrix} 3 & -5 \end{bmatrix}^T$
12. Determine whether $\{\bar{x}, \bar{y}, \bar{z}\}$ is an orthogonal set of vectors in $\mathbb{R}^3$ where $\bar{x} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \bar{y} = \begin{bmatrix} 1 \\ 1 \\ -2 \end{bmatrix}, \bar{z} = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$?

Part B

II. Answer any Six questions. Each question carries 5 marks **(6x5=30)**

13. Prove that if a set of vectors S in a vector space V is linearly dependent, then any larger set containing S is also linearly dependent.
14. Determine whether $\left\{\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix}\right\}$ is linearly independent.
15. Prove that if $\{\bar{u}, \bar{v}\}$ is linearly independent, then so too is $\{\bar{u} + \bar{v}, \bar{u} - \bar{v}\}$.

16. Prove that a linear transformation is one-to-one if and only if the kernel contains just the zero vector.

17. Find the transition matrix from $B = \{\begin{bmatrix}1 & 0\end{bmatrix}^T, \begin{bmatrix}1 & 1\end{bmatrix}^T\}$ to $C = \{\begin{bmatrix}0 & 1\end{bmatrix}^T, \begin{bmatrix}1 & 1\end{bmatrix}^T\}$.

18. Let a linear transformation $T : V \mapsto W$ have the property that the dimension of V equals the dimension of W. Then prove that T is one-to-one if and only if T is onto.

19. Establish that similar matrices have the same characteristic equation.

20. Prove that if $\bar{x}$ is an eigen vector of A corresponding to the eigen value λ, then establish that for any scalar c, $\lambda - c$ and $\bar{x}$ are a corresponding pair of eigen values and eigen vectors of $A - cI$.

21. Establish that if $\bar{x}$ and $\bar{y}$ are orthogonal vectors in $\mathbb{R}^n$, then $||\bar{x} + \bar{y}||^2 = ||\bar{x}||^2 + ||\bar{y}||^2$

Part C

III. Answer any Two questions. Each question carries 15 marks **(2x15=30)**

22. Describe the span of the vectors in the set $\mathbb{R} = \{\begin{bmatrix}1 & 1\\0 & 0\end{bmatrix}, \begin{bmatrix}0 & 1\\0 & 1\end{bmatrix}, \begin{bmatrix}1 & 0\\0 & -1\end{bmatrix}, \begin{bmatrix}0 & 0\\1 & -1\end{bmatrix}, \begin{bmatrix}0 & 1\\1 & 0\end{bmatrix}\}$

23. Calculate the transition matrix from $S = \{\begin{bmatrix}1 & 0 & 0\end{bmatrix}^T, \begin{bmatrix}0 & 1 & 0\end{bmatrix}^T, \begin{bmatrix}0 & 0 & 1\end{bmatrix}^T\}$ to $T = \{\begin{bmatrix}1 & 1 & 0\end{bmatrix}^T, \begin{bmatrix}0 & 1 & 1\end{bmatrix}^T, \begin{bmatrix}1 & 0 & 1\end{bmatrix}^T\}$

24. Define nullity and rank of a linear transformation. Find the nullity and rank of the linear transformation $T : \mathbb{R}^2 \mapsto \mathbb{R}^3$ defined by $T\begin{bmatrix}a & b\end{bmatrix} = \begin{bmatrix}a+b & 2a+b & a\end{bmatrix}$ and determine whether it is one-to- one and onto.

25. Find the eigen values and eigen vectors of $A = \begin{bmatrix}2 & -1 & 0\\3 & -2 & 0\\0 & 0 & 1\end{bmatrix}$