TB206500W

Reg. No :...............................

Name :.................................

**B.Sc. DEGREE (C.B.C.S.) EXAMINATION, MARCH 2023**

**(2020 Admission Regular, 2019, 2018 Admissions Supplementary)**

**SEMESTER VI - CORE COURSE (MATHEMATICS)**

**MT6B10B18 - COMPLEX ANALYSIS**

**Time : 3 Hours** **Maximum Marks : 80**

## Part A

**I. Answer any Ten questions. Each question carries 2 marks** **(10x2=20)**

1. Split into real and imaginary parts. $f(z) = z + \frac{1}{z}$.
2. Define closure of a set S.
3. Evaluate $\lim_{z \to i} \frac{iz^3 - 1}{z + i}$.
4. Using the formal definition of a derivative evaluate $f'(z)$, where $f(z) = z^2$.
5. Define a domain.
6. Evaluate $\int_0^1 (1 + it)^2 dt$.
7. Evaluate $\int_C \bar{z} dz$ where C is the right hand semi circle of $|z| = 2(-\pi/2 \leq \theta \leq \pi/2)$.
8. Define a simply connected domain. Is the region between two concentric circles simply connected or multiply connected?
9. Find the series representation of $\frac{1}{1-z}.(|z| < 1)$
10. Show that when $z \neq 0$, $\frac{\sin z^2}{z^4} = \frac{1}{z^2} - \frac{z^2}{3!} + \frac{z^6}{5!} - ...$
11. Find the order of the zero of $f(z) = z(e^z - 1)$ at z=0.
12. Show that z=0 is a pole of order 2 for $f(z) = \frac{1}{z^2(1+z)}$.

## Part B

**II. Answer any Six questions. Each question carries 5 marks** **(6x5=30)**

13. Evaluate $\int_0^{\pi/4} e^{it} dt$.
14. Show that if v and V are harmonic conjugates of u in a domain D, then v and V can differ at most by an additive constant
15. Identify the points where f(z) fails to be analytic. a) $f(z) = \frac{2z+1}{z(z^2+1)}$ b) $f(z) = \frac{z^3 + i}{z^2 - 3z + 2}$.
16. Evaluate $\int_C f(z)dz$ where $f(z) = \frac{z^2+1}{z^2-1}$ where . $c: |z| = 2.$
17. State and prove ML inequality theorem.

18. Evaluate $\int_C \frac{z^2-1/3}{z^3-z}dz$ where C is $|z-1/2|=1$

19. Show that when $z\neq 0$ $\frac{\sin hz}{z^2} = \frac{1}{z} + \sum_{n=0}^{\infty}\frac{z^{2n+1}}{(2n+3)!}$ $0<|z|<\infty$.

20. Write the principal part of $\frac{z^2}{1+z}$ at its isolated singular point and determine whether the point is a pole , a removable singularity, or an essential singular point.

21. Find the order of the pole and its residue at z=2 of $\frac{z^2-2z+3}{z-2}$.

**Part C**

**III. Answer any Two questions. Each question carries 15 marks** **(2x15=30)**

22. a. Evaluate $i^{-2i}$.

b.Show that $\sin^{-1}z = -i\log[iz+(1-z^2)^{1/2}]$.

c.Find the solutions of $e^z=3$.

d. Evaluate log(-1).

e.Solve $e^z = 1+i$.

23. If $f(z)$ is an analytic function inside and on a closed contour C described in the positive sense and $z_0$ is an interior point of C ,then prove that $f^n(z_0) = \frac{n!}{2\pi i}\int_c \frac{f(z)}{(z-z_0)^{n+1}}dz$.

24. a. Find the expansion of the function $f(z) = \frac{5z-2}{(z-1)(z-2)}$ in the region.

(i) $|z|<1$ (ii) $1<|z|<2$ (iii) $|z|>2$.

b. Find the Laurent's series expansion of $f(z) = \frac{1+z^2}{z^2+z}$ in the region 0<|z|<1

25. Use residue to evaluate $\int_0^{\infty}\frac{x^2dx}{x^6+1}$.