TB206275W **Reg. No :...............................**

Name :..................................

B. Sc. DEGREE (C.B.C.S) EXAMINATION, MARCH 2023

(2020 Admission Regular, 2019, 2018 Admissions Supplementary)

SEMESTER VI - CORE COURSE (MATHEMATICS)

MT6B09B18 - REAL ANALYSIS –II

(Common for Mathematics and Computer Applications)

Time : 3 Hours **Maximum Marks : 80**

Part A

I. Answer any Ten questions. Each question carries 2 marks **(10x2=20)**

1. Prove that the series $\sum r^n$ converges if 0 < r <1
2. Give an example of an infinite series for which Raabe's test fails. Justify your answer.
3. Give an example of a diverging series $\sum U_n$ with $\lim_{n \to \infty} \frac{U_{n+1}}{U_n} = 1$
4. Define uniform continuity with the help of an example.
5. Prove that difference of two continuous functions is continuous.
6. If f and g are functions continuous at a point c, then prove that (f + g) is continuous at c.
7. Define lower integral of a bounded function f over [a,b]
8. Define lower Darboux sum of a bounded function f over [a,b]
9. Find the upper integral of the function $f(x) = x^2$ over [0, 4].
10. Compute the lower Darboux sum L(P, f) of the function f(x) = 2x + 3 for the partition P = {2, 2.2, 2.9, 3} of [2, 3].
11. State a test using for checking the uniform convergence of a series of functions.
12. Check the uniform convergence of the series whose n^{th} term is $(1.8)^n \sin(n^2 x)$ for all real values of x.

Part B

II. Answer any Six questions. Each question carries 5 marks **(6x5=30)**

13. State and prove limit form of comparison test.
14. Check the convergence of the series whose n^{th} term is given by $(n^3+1)^{1/3} - n$
15. A function continuous on a closed interval [a, b] is uniformly continuous on [a, b]. Is the converse true?
16. Discuss the continuity of the function f defined on R by

$$f(x) = \begin{cases} -x^2, & \text{if } x \le 0 \\ 5x - 4, & \text{if } 0 < x \le 1 \\ 4x^2 - 3x & \text{if } 1 < x < 2 \\ 3x + 4 & \text{if } x \ge 2 \end{cases}$$

at the points x = 0, 1 and 2 .

17. Prove or disprove : If | f | is bounded and integrable on [a, b], then f is bounded and integrable on [a, b].
18. If f and g are bounded and integrable functions on [a,b] such that $f \ge g$, then prove that

$$\int_a^b f\,dx \ \ge \int_a^b g\,dx \ ; b \ge a$$

19. If a refinement P^* of the partition P of [a, b] contains p points more than P, and $f(x) \leq k \ \forall x \in [a, b]$, then prove that $L(P, f) \leq L(P^*, f) \leq L(P, f) + 2pk\mu$ where μ is the norm of P.

20. State and prove Weierstrass M- test for uniform convergence of a series of functions.

21. Show that $\{ f_n \}$ where $f_n(x) = \tan^{-1} nx$, x > 0 is uniformly convergent in [1, 2].

Part C

III. Answer any Two questions. Each question carries 15 marks **(2x15=30)**

22. Define an alternating series. State and prove the Leibnitz test for checking the convergence of an alternating series.

23. (a) If a function f is continuous on [a,b] and f(a).f(b) < 0, then prove that there exist at least one point c in (a, b) such that f(c) = 0.

    (b) Show that the function f(x) defined on R by

$$f(x) = \begin{cases} \text{x} \ , \text{ if } \ x \ is \ irrational \\ -\text{x}, \ \text{ if x is rational} \end{cases}$$

    is continuous only at x=0.

24. (a) State and prove Darboux's theorem.

    (b) Prove that, a function is integrable over [a, b] iff there is a number I lying between L(P, f) and U(P, f) such that for every $\varepsilon > 0$, there exist a partition P of [a, b] such that $| U(P, f) - I | < \varepsilon$ and $| I - L(P, f) | < \varepsilon$

25. a) State and prove Weierstrass M- test for uniform convergence of a series of functions

    b) Define Pointwise convergence and Uniform convergence with the help of an example.