TB256151Z

Reg. No :...............................

Name :.................................

BACHELOR'S DEGREE (C.B.C.S) EXAMINATION, MARCH 2025

2018, 2019, 2020, 2021 ADMISSIONS SUPPLEMENTARY

SEMESTER VI - CORE COURSE (MATHEMATICS)

MT6B12B18 - Linear Algebra

Time : 3 Hours **Maximum Marks : 80**

Part A

I. Answer any Ten questions. Each question carries 2 marks **(10x2=20)**

1. Give an example of a two dimensional vector space.
2. Determine whether $S = \{p(t) \in \mathbb{P}^2 | p(2) = 1)\}$ is a subspace of $\mathbb{P}^2$?
3. Define similar matrices.
4. Determine whether the transformation L is linear if $L : \mathbb{R}^2 \mapsto \mathbb{R}^2$ is defined by $L\begin{bmatrix} a & b \end{bmatrix} = \begin{bmatrix} a & 0 \end{bmatrix}$ for all real numbers a and b.
5. A linear transformation $T : \mathbb{R}^2 \mapsto \mathbb{R}^2$ has the property that $T\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 5 \\ 6 \end{bmatrix}$ and $T\begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} 7 \\ 8 \end{bmatrix}$. Determine $T(\bar{v})$ for any vector $\bar{v} \in \mathbb{R}^2$.
6. Define eigen vector of a linear transformation.
7. Define characteristic equation of a matrix A.
8. Calculate $< A, B >$ for $A = \begin{bmatrix} 4 & 3 \\ 6 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 \\ 1 & 2 \end{bmatrix}$ in the vector space $M_{2\times2}$ with respect to the standard basis.
9. A 3x3 matrix A is known to have the eigen values -2, 2 and 4. Calculate the eigen values of $A + 3I$.
10. State the generalized theorem of Pythagoras.
11. Determine whether $\bar{x} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ and $\bar{y} = \begin{bmatrix} -3 \\ -6 \\ 5 \end{bmatrix}$ are orthogonal ?
12. Determine whether $\{\bar{x}, \bar{y}, \bar{z}\}$ is an orthogonal set of vectors in $\mathbb{R}^3$ where $\bar{x} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \bar{y} = \begin{bmatrix} 1 \\ 1 \\ -2 \end{bmatrix}, \bar{z} = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$?

Part B

II. Answer any Six questions. Each question carries 5 marks **(6x5=30)**

13. Establish that if a set of vectors S in a vector space V is linearly independent, then any subset of S is also linearly independent.
14. For any vector $\bar{w}$ in a vector space V, prove that $-(-\bar{w}) = \bar{w}$
15. In any vector space V, prove that $\alpha \odot \bar{0} = \bar{0}$, for every scalar α.
16. Establish that every linear transformation can be represented by a matrix.
17. Find $T(\bar{u})$ for a linear transformation T if it is known that $T(\bar{u} + \bar{v}) = 2\bar{u} + 3\bar{v}$ and $T(\bar{u} - \bar{v}) = 4\bar{u} + 5\bar{v}$

18. Find the transition matrices between two bases $G = \{t+1, t-1\}$ and $H = \{2t+1, 3t+1\}$ for $\mathbb{P}^1$

19. Prove that the eigen vectors of a matrix corresponding to distinct eigen values are linearly independent.

20. If $\bar{x}$ is an eigen vector of A corresponding to the eigen value λ then prove that λ^n and $\bar{x}$ are a corresponding pair of eigen values and eigen vectors of A^n, for any positive integer n.

21. Establish that if $\bar{x}$ and $\bar{y}$ are orthogonal vectors in $\mathbb{R}^n$, then $||\bar{x}+\bar{y}||^2 = ||\bar{x}||^2 + ||\bar{y}||^2$

Part C

III. Answer any Two questions. Each question carries 15 marks **(2x15=30)**

22. Let S be a non empty subset of a vector space V with operations $\oplus$ and $\odot$. Prove that S is a subspace of V if and only if the following two closure conditions hold: a) Closure under addition and b) Closure under scalar multiplication.

23. Calculate the transition matrix from $S = \{\begin{bmatrix}1 & 0 & 0\end{bmatrix}^T, \begin{bmatrix}0 & 1 & 0\end{bmatrix}^T, \begin{bmatrix}0 & 0 & 1\end{bmatrix}^T\}$ to $T = \{\begin{bmatrix}1 & 1 & 0\end{bmatrix}^T, \begin{bmatrix}0 & 1 & 1\end{bmatrix}^T, \begin{bmatrix}1 & 0 & 1\end{bmatrix}^T\}$

24. Find a basis for the kernel and image of the matrix $A = \begin{bmatrix}1 & 1 & 1\\1 & 1 & 1\\1 & 1 & 1\end{bmatrix}$

25. Find the eigen values and eigen vectors of $A = \begin{bmatrix}2 & -1 & 0\\3 & -2 & 0\\0 & 0 & 1\end{bmatrix}$