

TB25033S

Reg. No.....

Name.....

B. Sc. DEGREE (C.B.C.S.S) EXAMINATION, MARCH 2025

(2016 and 2017 Admissions Supplementary)

SEMESTER VI – CORE COURSE (MATHEMATICS)

MT6B09B- REAL ANALYSIS II

Time: 3 Hours

Maximum Marks: 80

PART A

I. Answer all questions. Each question carries 1 mark

1. Give an example of a sequence of functions whose pointwise limit is zero
2. Define removable discontinuity
3. Define uniform convergence of a sequence of functions and give an example.
4. State Cauchy's general principle of convergence for infinite series.
5. Define lower integral of a bounded function f over $[a,b]$
6. State D'Alembert's Ratio test.

(6x1=6)

PART B

II. Answer any seven questions. Each question carries 2 marks

7. State the intermediate value theorem
8. Define uniform continuity with the help of an example.
9. Define upper integral of a bounded function f over $[a,b]$.
10. Show that a constant function k is integrable over any interval
11. State Dirichlet's test for uniform convergence
12. Define absolute convergence of an infinite series with the help of an example
13. Prove that sum of two continuous functions is continuous.
14. State limit form of comparison test for checking the convergence of a positive infinite series.
15. State Cauchy's root test for checking the convergence of a positive infinite series.
16. Define pointwise convergence of a sequence of functions. Give an example

(7x2=14)

PART C

III. Answer any 5 questions. Each question carries 6 marks

17. State and prove Fixed point theorem
18. Prove that every absolutely convergent series is convergent
19. If a function f is monotonic on $[a,b]$, then show that it is integrable on $[a,b]$.
20. State and prove Darboux's theorem.
21. State and prove Cauchy's criterion for uniform convergence of a series of functions
22. State and prove Raabe's test

23. Prove that a function which is continuous on a closed interval is also uniformly continuous on that interval.
24. State and prove a test using for checking the uniform convergence of a sequence of functions.

(5x6=30)

PART D

IV. Answer any 2 questions. Each question carries 15 marks

25. State and prove a necessary and sufficient condition for the continuity of a function at a point in an interval
26. State and prove a test for (a) checking the uniform convergence of a series of functions.
(b) checking the uniform convergence of a sequence of functions.
27. Define an alternating series. State and prove the Leibnitz test for checking the convergence of an alternating series.
28. Prove that, a bounded function is integrable on $[a, b]$ iff for every $\varepsilon > 0$, there exist a partition P of $[a, b]$ such that $U(P, f) - L(P, f) < \varepsilon$.

(2x15=30)