TB256101U

Reg. No :................................

Name :..................................

BACHELOR'S DEGREE (C.B.C.S) EXAMINATION, MARCH 2025

2018, 2019, 2020, 2021 ADMISSIONS SUPPLEMENTARY

SEMESTER VI - CORE COURSE (MATHEMATICS)

MT6B09B18 - Real Analysis –II

Time : 3 Hours **Maximum Marks : 80**

Part A

I. Answer any Ten questions. Each question carries 2 marks **(10x2=20)**

1. State Cauchy's general principle of convergence for infinite series.
2. Determine whether the series $\sum \frac{n^2+4n+1}{n-3}$ converges or diverges.
3. Give an example of a diverging series $\sum U_n$ with $\lim_{n\to\infty} U_n = 0$
4. Prove that product of two continuous functions is continuous.
5. Is the function f(x) = |x| continuous ? Justify.
6. Show that the function $f(x) = x^2$ is uniformly continuous on [-2, 2]
7. Is Dirichlet's function Riemann integrable over a closed interval [a, b] ? justify your answer.
8. Define upper Darboux sum of a bounded function f over [a,b]
9. If P^* is a refinement of partition P, the show that $U(P^*, f) \leq U(P, f)$.
10. Find the upper sum U(P, f) of the function $f(x) = \begin{cases} 0, & \text{when x is rational} \\ 1, & \text{when x is irrational} \end{cases}$ corresponding to the partition P = {1, 1.5, 2} of the interval [1, 2]
11. State a test using for checking the uniform convergence of a series of functions.
12. Check the uniform convergence of the series whose n^{th} term is $(0.5)^n \cos n^2 x$.

Part B

II. Answer any Six questions. Each question carries 5 marks **(6x5=30)**

13. State and prove D'Alembert's Ratio test.
14. Investigate the nature of convergence of the series $\sum \frac{n^2-1}{n^2+1} x^n , x > 0$
15. Prove that the range of a continuous function whose domain is a closed interval is as well a closed interval.
16. If a function f is continuous at an interior point c of [a, b] and f(c) is a non zero real number, then prove the existence of a neighborhood of c where f(x) has the same sign as f(c).
17. If a function f is continuous on [a,b], then show that it is integrable on [a,b].
18. Prove that, if f is bounded and integrable on [a, b], and $f(x) \geq 0 \ \forall x \in [a, b]$, then $$\int_a^b f dx \begin{cases} \geq 0, & if\ b \geq a \\ \leq 0, & if\ b \leq a \end{cases}$$
19. State and prove Darboux's theorem.
20. State and prove a test for uniform convergence of a series of functions.
21. Show that $\{ f_n \}$ where $f_n(x) = \frac{1}{x+n}$ is uniformly convergent in [0, 1]

Part C

III. Answer any Two questions. Each question carries 15 marks **(2x15=30)**

22. State and prove Leibnitz test for checking the convergence of an alternating series
23. Prove that a function which is continuous on a closed interval [a, b] is bounded and attains its bounds at least once in [a, b].
24. State and prove the first and the second Fundamental Theorems of integral calculus.
25. (a)State and prove a test for checking the uniform convergence of a sequence of functions.

    (b) Show that the sequence $\{f_n\}$ where $f_n(x) = nxe^{-nx^2}$, $x \geq 0$ is not uniformly convergent on [0, 4].