TB254817B

Reg. No :...............................

Name :...................................

BACHELOR'S DEGREE (C.B.C.S) EXAMINATION, MARCH 2025

2018, 2019, 2020, 2021, 2022 ADMISSIONS SUPPLEMENTARY

SEMESTER IV - CORE COURSE (MATHEMATICS)

MT4B04B18 - Vector Calculus, Theory of Equations and Matrices

Time : 3 Hours **Maximum Marks : 80**

Part A

I. Answer any Ten questions. Each question carries 2 marks **(10x2=20)**

1. Define a binormal vector.
2. A particle moves along the curve $x=3t^2,\ y=t^2-2t,\ z=t^3$. Find the velocity at t =1.
3. Find an equation for the plane through P(2,4,5) and perpendicular to the line $x=5+t,\ y=1+3t,\ z=4t$.
4. Define the flux of a vector field.
5. Find a parametrization of the cone $z=\sqrt{x^2+y^2}, 0\le z\le 1$
6. Determine whether $\bar{\mathbf{F}}=(2x-3)\bar{i}-z\bar{j}+(\cos z)\bar{k}$ is conservative.
7. Define reciprocal equation. Give an example of a standard reciprocal equation.
8. If α, β, γ are the roots of $x^3+px^2+qx+r=0$ then find the value of $\frac{1}{\alpha}+\frac{1}{\beta}+\frac{1}{\gamma}$
9. Find the condition that the roots of the equation $x^3+px^2+qx+r=0$ may be in Geometric Progression
10. Define the Normal form of a matrix
11. Find the rank of the matrix $\begin{bmatrix}1 & 2 & 3\\ 2 & 4 & 5\end{bmatrix}$
12. Find the rank of the matrix $\begin{bmatrix}1 & 2 & 3\\ 2 & 3 & 4\\ 3 & 5 & 7\end{bmatrix}$.

Part B

II. Answer any Six questions. Each question carries 5 marks **(6x5=30)**

13. Find the distance from (2,-3,4) to the plane $x+2y+2z=13$.
14. Find the unit tangent vector to the curve $x=t^2+1,\ y=4t-3,\ z=2t^2-6t$ at the point t=2.
15. State both forms of Green's theorem
16. Find the surface area of a sphere of radius a.
17. Show that $\bar{F}=(e^x\cos y+yz)\bar{i}+(xz-e^x\sin y)\bar{j}+(xy+z)\bar{k}$ is conservative and find a potential function for it.
18. Prove that every polynomial equation of nth degree has exactly n roots
19. Solve the equation $x^4+2x^3-25x^2-26x+120=0$ given that the product of two of its roots is 8
20. Test for consistency and then solve
$x+2y-5z=-93x-y+2z=52x+3y-z+34x-5y+z=-3$.

21. Obtain the row equivalent canonical matrix of $A = \begin{bmatrix} 1 & 1 & 1 & 2 \\ 2 & 1 & -3 & -6 \\ 3 & -3 & 1 & 2 \end{bmatrix}$.

Part C

III. Answer any Two questions. Each question carries 15 marks **(2x15=30)**

22. Find T, N, B, κ, τ for the space curve $r(t) = \frac{t^3}{3}\vec{i} + \frac{t^2}{2}\vec{j}$, $t > 0$.

23. Find the area of the surface cut from the bottom of the paraboloid $x^2 + y^2 - z = 0$ by the plane $z = 4$

24. Solve $6x^5+11x^4-33x^3-33x^2+11x+6 = 0$

25. Show that the system of equations $x + 2y + z = 23x + y - 2z = 14x - 3y - z = 32x + 4y + 2z = 4$ is consistent and hence solve the same.