TB254178L

Reg. No :..............................

Name :..............................

BACHELOR'S DEGREE (C.B.C.S) EXAMINATION, MARCH 2025

2018, 2019, 2020, 2021, 2022 ADMISSIONS SUPPLEMENTARY

SEMESTER IV - COMPLEMENTARY COURSE 1 (MATHEMATICS)

MT4C01B18 - Fourier Series, Partial Differential Equations, Numerical Analysis and Abstract Algebra

Time : 3 Hours **Maximum Marks : 80**

Part A

I. Answer any Ten questions. Each question carries 2 marks **(10x2=20)**

1. Define odd functions.
2. Verify that $P_0(x) = 1$
3. Show that $P_3(x) = \frac{1}{2}(5x^3 - 3x)$
4. Write the equation to find the tangent plane at a point P (x,y,z) to the surface S whose equation is F(x,y,z)=0.
5. Form a partial differential equation by eliminating the constants a and b from the equation $2z = (ax + y)^2 + b$.
6. Find the equation of the tangent plane to the surface $x + y + z = 0$ at the point $(\frac{1}{\sqrt{14}}, \frac{2}{\sqrt{14}}, \frac{-3}{\sqrt{14}})$.
7. Write Newton Raphson formula for finding an approximate root of f(x)=0
8. Does there exist a root between 2 and 3 for the equation x(sin x) = 2.5 ? Justify your answer.
9. Define Symmetric group on n letters
10. Define subgroup of a group G. Give an example.
11. Find the order of 5 in Z_6
12. Find the order of i and -i in the group {1, -1, i, -i} under the operation usual multiplication.

Part B

II. Answer any Six questions. Each question carries 5 marks **(6x5=30)**

13. Show that $J_{\frac{3}{2}}(x) = \sqrt{\frac{2}{\pi x}}\left(\frac{sin x}{x} - cos x\right)$.
14. Solve using power series method: $(1 + x)y' = y$.
15. Find a general solution of the differential equation $xy'' + 5y' + xy = 0$, in terms of J_v and J_{-v}. $(Hint : set\ y = \frac{u}{x^2})$.
16. Show that $J_1'(x) = J_0(x) - x^{-1}J_1(x)$.
17. Find the integral curves of the equations $\frac{dx}{z} = \frac{dy}{-z} = \frac{dz}{z^2 + (x + y)^2}$.
18. Find the positive root of the equation $x^2 - 4x + 3 = 0$ using bisection method.
19. If G is a group with binary operation $*$ and if a and b are any two elements of G, show that the linear equations $a * x = b$ and $y * a = b$ have unique solution in G.
20. Define a skew field . Check whether $< \mathbb{Z}, +, . >$ is a skew field where '+' is the usual addition, ' . ' is the usual multiplication and $\mathbb{Z}$ is the set of integers.

21. Define Kernel of a group Homomorphism. Find the kernel of the homomorphism $g: \mathbb{Z} \to \mathbb{R}$ under addition defined by g(x) = x.

Part C

III. Answer any Two questions. Each question carries 15 marks **(2x15=30)**

22. (a) Using the values $J_1(2) = 0.5767,\ J_0(2) = 0.2239,\ J_1(1) = 0.4401,\ J_0(1) = 0.7652,$ evaluate $I = \int_1^2 x^{-3} J_4(x).$

(b) Find a general solution of the differential equation $xy'' - 5y' + xy = 0\ (Hint: put y = x^3 u).$

23. (a) Find the integral curves of $\dfrac{dx}{x^2(y^3 - z^3)} = \dfrac{dy}{y^2(z^3 - x^3)} = \dfrac{dz}{z^2(x^3 - y^3)}$

(b) Eliminate the arbitrary function f from the eqution $z = xf(2x - y) + g(2x - y).$

24. Obtain a root of the equation $x^3 - 18 = 0$ correct to three decimal places using bisection method

25. (a). Define $*$ on Q^+ by $a * b = \dfrac{ab}{2}$. Show that Q^+ under the operation $*$ is an abelian group.

(b). Show that the set G ={1,-1, i, -i} forms an abelian group with respect to multiplication.