TB174465D

Reg. No: ....

Name: ....

B. Sc. DEGREE (C.B.C.S.S) EXAMINATION, MARCH 2025
(2017 & 2016 Admissions Supplementary)
SEMESTER IV - CORE COURSE (STATISTICS)
CAS4B04TB – STATISTICAL INFERENCE
(For Computer Applications)

Time: Three Hours **Maximum Marks: 80**

PART A

I Answer all questions. Each question carries 1 mark

1. Distinguish between a Statistic and a Parameter.
2. If T is an unbiased estimate of θ, examine whether T^2 is unbiased for θ^2.
3. What is the distribution of the ratio of two χ^2 variates?
4. Give a sufficient set of conditions for an estimate to be consistent.
5. What are the two types of errors in testing of hypotheses?
6. What is meant by a Statistical hypothesis?

(6x1=6)

PART B

II Answer any seven questions. Each question carries 2 marks

7. Define F statistic. Give an example of an F statistic.
8. State Crammer-Rao inequality for the variance of any unbiased estimate of a parameter.
9. Define efficiency of an estimate. Compare the efficiency of $T_1 = X_1$ and $T_2 = \frac{X_1 + X_2}{2}$ in estimating μ, where X_1, X_2 is a random sample from Normal population $N(\mu,\sigma)$
10. Explain the terms Type I error and Type II error.
11. Find the M.L. estimate of λ based on a sample taken from the population with p.d.f.
$f(x) = \frac{e^{-\lambda}\lambda^x}{x!}$, x = 0, 1, 2, ...
12. Which are the main steps in solving a testing of hypothesis problem?
13. State Neyman- Pearson lemma.
14. Define null hypothesis and alternative hypothesis.
15. Establish the relationship between t distribution and normal distribution.
16. Given the following contingency table, what is the value of the χ^2 statistic?

	A_1	A_2
B_1	a	b
B_2	c	d

(7x2=14)

PART C

III Answer any five questions. Each question carries 6 marks

17. If $X_1, X_2, \ldots, X_n$ are i.i.d. random variables following N(0,1), show that $\sum_{i=1}^{n} X_i^2$ follows χ^2 distribution with n degrees of freedom.
18. State the Cramer-Rao inequality. Examine whether the parameter λ of a Poisson distribution admits a minimum variance unbiased estimator. Also find the lower bound for the variance of any unbiased estimator of λ.
19. Obtain a 100(1 – α) % confidence interval for the variance of a Normal population using a sample of size n (n < 30) drawn from the population.
20. Obtain the 95% confidence interval for difference between the means of two normal populations, when the population standard deviations are known.
21. A sample of size 25 is drawn from a Normal population has variance 6.25.Find k such that $P[\bar{X} - \mu < k] = 0.60$ where $\bar{X}$ is the sample mean and μ is the population mean.
22. The sales data of an item in six shops before and after a special promotional campaign are as follows:

Shops	A	B	C	D	E	F
Before campaign	53	28	31	48	50	42
After campaign	58	29	30	55	56	45

Can the campaign be judged as a success?

23. Show that for the normal distribution with zero mean and variance σ^2, the best critical region for H$_0$: $\sigma = \sigma_0$ against the alternative H$_1$: $\sigma = \sigma_1$ is of the form $\sum_{i=1}^{n} X_i^2 \le a_\alpha$ for $\sigma_0 > \sigma_1$ and $\sum_{i=1}^{n} X_i^2 \ge b_\alpha$ for $\sigma_0 \le \sigma_1$.
24. Explain the procedure of testing equality of means when samples are drawn from normal population (considering all the cases).

(5x6=30)

PART D

III Answer any two questions. Each question carries 15 marks

25. (a) Derive the distribution of $t = \dfrac{\bar{x}_1 - \bar{x}_2 - (\mu_1 - \mu_2)}{\sqrt{\dfrac{n_1 s_1^2 + n_2 s_2^2}{n_1 + n_2 - 2}\left(\dfrac{1}{n_1} + \dfrac{1}{n_2}\right)}}$, where $\bar{x}_1$ and $\bar{x}_2$ are the means, s_1^2 and s_2^2 are the variances of random samples of sizes n_1 and n_2 from two Normal populations with the same unknown variance σ^2 .

(b) The following data pertain to the results of a memory retention test on the ability to remember the names of 10 objects of two groups of children:

Group A:	8	6	5	7	4
Group B:	10	6	7	8	9

If $\bar{x}_1$ and $\bar{x}_2$ are the sample means of Group A and Group B respectively, find P [$|\bar{x}_1 - \bar{x}_2| < 1$]

26 Show that sample mean is a sufficient estimate of the population mean μ if population standard deviation is known, but sample variance is not sufficient for population variance if population mean is known in sampling from $N(\mu,\sigma^2)$

27. (a) State the Neyman-Pearson lemma.

(b) Suppose a random sample of size n is drawn from a Poisson population with parameter λ, obtain the most powerful critical region for testing H_0: $\lambda=\lambda_0$ against H_1: $\lambda=\lambda_1$ when $\lambda_1 > \lambda_0$.

28. a) For the data in the following table, test for independence between a person's ability in Mathematics and interest in Economics.

		Ability in Maths		
		Low	Avg.	High
Interest in Economics	Low	63	42	15
	Avg.	58	61	31
	High	14	47	29

b) In a random sample of 500 men from a particular district of a state,300 are found to be smokers. In one of 1000 men from another district 550 are smokers. Do the data indicate that two districts are significantly different w.r.t the prevalence of smoking among men?

(2x15=30)