TB252265C

Reg. No :...............................

Name :...............................

BACHELOR'S DEGREE (C.B.C.S) EXAMINATION, MARCH 2025

2023 ADMISSIONS SUPPLEMENTARY

SEMESTER II - COMPLEMENTARY COURSE 1

MT2B01B23 - Partial Derivatives, Multiple Integrals Trigonometry and Matrices

Time : 3 Hours **Maximum Marks : 80**

Part A

I. Answer any Ten questions. Each question carries 2 marks **(10x2=20)**

1. State Fubini's Theorem (first form).
2. Calculate the volume under the plane $z = 4 - x - y$ over the rectangular region $R: 0 \le x \le 2, 0 \le y \le 1$ in the xy plane.
3. Change the Cartesian integral $\int_0^6 \int_0^y x\,dx dy$.
4. Prove that $cosh\ 3x = 4cosh^3\ x - 3\ cosh\ x$.
5. Show that $tanh\ (x - y) = \dfrac{tanh\ x - tanh\ y}{1 - tanh\ x \tanh\ y}$
6. Find the real and imaginary part of $sinh(A - iB)$.
7. Write the Chain Rule for Two Independent Variables and Three Intermediate Variables
8. Find f_x, f_y of $f(x, y) = (2x - 3y)^3$
9. Find $\dfrac{\partial f}{\partial x}$ and $\dfrac{\partial f}{\partial y}$, if $f(x, y) = (x^2 - 1)(y - 2)$.
10. Write the characteristic equation of the matrix $A = \begin{bmatrix} 1 & 2 \\ 3 & 1 \end{bmatrix}$.
11. Find the eigen values of the matrix $\begin{bmatrix} 1 & 2 & 3 \\ 0 & -4 & 2 \\ 0 & 0 & 7 \end{bmatrix}$.
12. Find the eigen values of the matrix $\begin{bmatrix} 3 & 4 \\ 5 & 2 \end{bmatrix}$

Part B

II. Answer any Six questions. Each question carries 5 marks **(6x5=30)**

13. Calculate the average value of $F(x, y, z) = x - y - z$ over the rectangular solid in the first octant bounded by the coordinate planes and the planes $x = 1, y = 1, z = 2$.
14. Sketch the region of integration and evaluate $\int_1^2 \int_y^{y^2} dxdy$.
15. Separate into real and imaginary parts the expression $tan^{-1}(x - iy)$.

16. If $sin(A+iB) = x+iy$, show that $\frac{x^2}{cosh^2 B} + \frac{y^2}{sinh^2 B} = 1$ and $\frac{x^2}{sin^2 A} - \frac{y^2}{cos^2 A} = 1$

17. Express $\frac{\partial w}{\partial r}, \frac{\partial w}{\partial s}$ in terms of r and s if $w = x + 2y + z^2,\ x = \frac{r}{s},\ y = r^2 + ln\ s,\ z = 2r.$

18. Show that the function $f(x, y, z) = x^2 + y^2 - 2z^2$ satisfies the equation $\frac{\partial^f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2} = 0$

19. Verify Caley Hamilton theroem for the matrix $\begin{bmatrix} 1 & 0 & 3 \\ 2 & 1 & -1 \\ 1 & -1 & 2 \end{bmatrix}.$

20. Reduce to the Echelon form and find the rank of the matrix $A = \begin{bmatrix} 1 & 6 & -18 \\ -4 & 0 & 5 \\ -3 & 6 & -13 \end{bmatrix}.$

21. Reduce to the Echelon form and find the rank of the matrix $A = \begin{bmatrix} 1 & -2 & 1 \\ 2 & 1 & 1 \\ 0 & 5 & -1 \end{bmatrix}.$

Part C

III. Answer any Two questions. Each question carries 15 marks **(2x15=30)**

22. (a) Find the average value $F(x, y, z) = xyz$ over the cube in the first octant bounded by the coordinate planes and the planes $x = 2, y = 2, z = 2.$

(b) Evaluate $\int_0^3 \int_0^{\sqrt{9-x^2}} \int_0^{\sqrt{9-x^2}} dz\ dy\ dx$.

(c) Integrate $f(x, y) = \frac{1}{xy}$ over the square $1 \le x \le 2, 1 \le y \le 2.$

23. Sum to infinity the series : $1 + a\ cos\theta + a^2 cos\ 2\theta + a^3 cos\ 3\theta + \ldots\ldots$ where $|a| < 1.$

24. (a) Evalaute the partial derivatives $\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}$ of the function $f(x, y) = 4 + 2x - 3y - xy^2$ at the point $(-2, 1)$.

(b) Evaluate $\frac{\partial w}{\partial u}, \frac{\partial w}{\partial v}$ at the point $(u, v) = (\frac{1}{2}, 1)$ given $w = xy + yz,\ x = u + v,\ y = u - v,\ z = 2u$

25. (a)Using Cayley Hamilton Theorem calculate A^4, if $A = \begin{bmatrix} -1 & 3 \\ -2 & 4 \end{bmatrix}$

(b) Solve completely the system of equations:

$$x - 2y - z - w = 0x + y - 2z - 3w = 04x - y - 5z + 8w = 05x - 7y + 2z - w = 0$$