TB252954M

Reg. No :..............................

Name :..............................

BACHELOR'S DEGREE (C.B.C.S) EXAMINATION, MARCH 2025

2018, 2019, 2020, 2021, 2022 ADMISSIONS SUPPLEMENTARY

SEMESTER II - COMPLEMENTARY COURSE 1 (MATHEMATICS)

MT2C01B18 - Partial Derivatives, Multiple Integrals Trigonometry and Matrices

Time : 3 Hours **Maximum Marks : 80**

Part A

I. Answer any Ten questions. Each question carries 2 marks **(10x2=20)**

1. State any two properties of double Integrals.
2. Write the polar equation of the circle $x^2 + y^2 = 1$.
3. Evalaute $\int_0^1 \int_0^{1-z} \int_0^2 dx dy dz$
4. If $cos(x + iy) = cos\theta + i\ sin\theta$, prove that $cos2x + cosh\ 2y = 2$.
5. Find the real and imaginary parts of $cos(x + iy)$.
6. Expand $sin^6\theta$ in series of cosines of multiples of θ.
7. Write the chain rule of differentiation for functions of two independent variables.
8. Find $\frac{\partial^2 f}{\partial x^2}$ of $f(x, y) = sin\ xy$.
9. Find f_{xy} of $f(x, y) = xsiny + e^y$.
10. Define equivalent matrices.
11. Find the rank of the matrix $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 0 & 2 & 2 \end{bmatrix}$.
12. Find the rank of the matrix $\begin{bmatrix} 2 & 4 & 3 & 2 \\ 3 & 3 & 1 & 4 \end{bmatrix}$.

Part B

II. Answer any Six questions. Each question carries 5 marks **(6x5=30)**

13. Evaluate using Polar coordinates : $\int\int_R e^{x^2-y^2}\ dydx$, where R is the semicircular region bounded by the $x - axis$ and the curve $y = \sqrt{1 - x^2}$.
14. Evaluate the area enclosed by the lemniscate $r^2 = 4\ cos2\theta$.
15. Show that $log\frac{sin(x + iy)}{sin(x - iy)} = 2i\ tan^{-1}(cot\ x.\ tanh\ y)$.
16. Sum to infinity the series : $1 + a\ cos\theta + a^2 cos\ 2\theta + a^3 cos\ 3\theta + \ldots\ldots$ where $|a| < 1$.
17. Show that the function $f(x, y, z) = x^2 + y^2 - 2z^2$ satisfies the equation $\frac{\partial f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2} = 0$
18. Express $\frac{\partial w}{\partial r}, \frac{\partial w}{\partial s}$ in terms of r and s if $w = x + 2y + z^2,\ x = \frac{r}{s},\ y = r^2 + ln\ s,\ z = 2r.$

19. Find the rank of the matrix by reducing to its normal form

$$A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 3 & 4 \\ 4 & 3 & 2 & 1 \end{bmatrix}$$

20. $$A = \begin{bmatrix} 7 & -2 & 2 \\ -2 & 1 & 4 \\ -2 & 4 & 1 \end{bmatrix}$$

Find the eigen values and corresponding eigen vectors of the matrix

21. Show that the system of linear equation is inconsistent:

$2x + 6y + 11 = 04x + 14y - z - 7 = 06y - 3z + 2 = 0.$

Part C

III. Answer any Two questions. Each question carries 15 marks **(2x15=30)**

22. (a) Find the average value $F(x, y, z) = xyz$ over the cube in the first octant bounded by the coordinate planes and the planes $x = 2, y = 2, z = 2.$

(b) Evaluate $\int_0^3 \int_0^{\sqrt{9-x^2}} \int_0^{\sqrt{9-x^2}} dz\, dy\, dx$.

(c) Integrate $f(x, y) = \frac{1}{xy}$ over the square $1 \leq x \leq 2, 1 \leq y \leq 2.$

23. (a) Derive the expansion of $tan(n\theta)$.

(b)Sum the series : $cos\ \alpha +^n C_1 cos\ (\alpha + \beta) +^n C_2\ cos(\alpha + 2\beta) + \ldots\ldots + cos\ (\alpha + n\beta)$.

24.

(a) Evalaute the partial derivatives $\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}$ of the function $f(x, y) = 4 + 2x - 3y - xy^2$ at the point $(-2, 1)$.

(b) Evaluate $\frac{\partial w}{\partial u}, \frac{\partial w}{\partial v}$ at the point $(u, v) = (\frac{1}{2}, 1)$ given $w = xy + yz + xz,\ x = u + v,\ y = u - v,\ z = uv.$

(c) Using implicit differentiation evaluate $\frac{\partial z}{\partial x}, \frac{\partial z}{\partial y}$ of the function $F(x, y, z) = z^3 - xy + yz + y^3 - 2 = 0$ at the point $(1, 1, 1)$.

25. Solve by Cramer's rule :

$$2a + b + 5c + d = 5a + b - 3c - 4d = -13a + 6b - 2c + d = 82a + 2b + 2c - 3d = 2$$