TB145600D

Reg No. ....

Name ....

B. Sc. DEGREE (C.B.C.S.S) EXAMINATION , NOVEMBER 2018
(2014 Admission Supplementary)
SEMESTER V - CORE COURSE (MATHEMATICS)
MAT5MA – MATHEMATICAL ANALYSIS

Time: Three Hours **Maximum Marks: 80**

PART A

I. Answer all questions. Each question carries 1 mark.

1. Find the infimumand supremum of the set $\left\{1+\frac{(-1)^n}{n}, n \in N\right\}$.
2. What is order completeness property of real numbers.
3. Define neighbourhood of a point.
4. State Bolzano-Weierstrass Theorem for sets.
5. When will you say that a set is perfect?
6. Define a limit point of asequence.
7. What do you mean by a Cauchy sequence?
8. State Cauchy's general principle of convergence.
9. Show that Im(iz)=Rez.
10. Express -1-i in exponential form.

(10x1=10)

PART B

II. Answer any eight questions. Each question carries 2 marks.

11. Define an ordered field.
12. Prove that the greatest number of a set if it exists is the supremum of the set.
13. Show that a set cannot have more than one supremum.
14. Show that the interior of the set N of natural numbers is the null set.
15. Show that the closure of a set is a closed set.
16. Show that the set of rational numbers in [0,1] is countable.
17. Show that every monotonic decreasing sequencewhich is not bounded below, diverges to $-\infty$
18. Show that every convergent sequence is bounded.
19. Show that $\lim_{n \to \infty} \sqrt[n]{n} = 1$.
20. Show that a sequence of real numbers can have at most one limit.
21. Sketch the set $|2z+3|>4$.
22. Prove that $|z_1+z_2|^2 \le |z_1|^2+|z_2|.^2$

(8x2= 16)

(P.T.O)

PART C

III. Answer any six questions. Each question carries 4 marks.

23. Show that the real number field is Archimedean.
24. Prove that every open interval]a,b[contains infinitely many rational numbers.
25. Show that every open set is the union of open intervals.
26. Prove that a set is closed iff its complement is open.
27. Show that derived set of a bounded set is bounded.
28. Use Cauchy's general principle of convergence to show that the sequence $\left\{\frac{n}{n+1}\right\}$ is convergent.
29. Show that every bounded sequence with a unique limit point is convergent.
30. Prove that a necessary and sufficient condition for the convergence of a monotonic sequence is that it is bounded.
31. Use De-Moivre's formula to derive $\sin 3\theta = 3\cos^2\theta \sin\theta - \sin^3\theta$.

(6x4= 24)

PART D

IV. Answer any two questions. Each question carries 15 marks.

32. Prove the equivalence of Dedekind's property and order completeness property of real numbers.
33. (a) Show that the interior of a set S is the largest open subset of S.
    (b)Show that the derived setS'of a bounded infinite set has the smallest and the greatest members.
34. State and prove nested interval property of real numbers.
35. State and prove Cauchy's first theorem on limits.

(2x15= 30)