TM252324X

Reg. No :..............................

Name :..............................

MASTER'S DEGREE (C.S.S) EXAMINATION, MARCH 2025

2020, 2021, 2022, 2023 ADMISSIONS SUPPLEMENTARY

SEMESTER II - CORE COURSE

ST2C07TM - Stochastic Processes

Time : 3 Hours **Maximum Weight : 30**

Part A

I. Answer any Eight questions. Each question carries 1 weight **(8x1=8)**

1. Define first passage transition probability.
2. State Basic limit theorem for a M.C.
3. Define inverse transition probability.
4. Explain one dimensional random walk.
5. Explain random walk in space.
6. If X(s) is a Poisson process with parameter λ, then show that P[X(s) = k/X(t) = n](where $t \geq s$) follows Binomial distribution.
7. Explain pure birth process.
8. Find the characteristic function of a poisson process.
9. Define Renewal function.
10. Give the generalized form of Renewal equation.

Part B

II. Answer any Six questions. Each question carries 2 weight **(6x2=12)**

11. Prove that unity is the characteristic root of any transition probability matrix.
12. For the three state Markov chain with TPM P = $\begin{bmatrix} 0.40 & 0.35 & 0.25 \\ 0.75 & 0.18 & 0.07 \\ 0.15 & 0.12 & 0.73 \end{bmatrix}$ Obtain the stationary distribution πk.
13. Let P(s) be the pgf associated with the offspring distribution P_k and $P_n(s)$ is asociated with X_n. Then show that $P_n(s) = P_{n-1}[P(s)]$.
14. If the p.g.f of offspring distribution is $P(s) = \frac{q}{1-p.s}$,$q > p$, $p+q$. Obtain the distribution of ultimate extinction of the process.
15. Explain the relationship between Poisson process and Uniform distribution.
16. Explain Kolmogrov's forward differential equation.
17. Derive renewal equation for delayed renewal process.
18. Show that the renewal function $M(t) = \sum_{n=1}^{\infty} F_n(x)$ Where $F_n(x) = P[S_n \leq x]$.

Part C

III. Answer any Two questions. Each question carries 5 weight **(2x5=10)**

19. (i) In usual notations, show that the state j is persistant iff $\sum p_{jj}^{(n)} = \infty$.

(ii) A state j is transient iff $\sum p_{jj}^{(n)} < \infty$.

(iii) A state j is recurrent if expected number of return is ∞.

20. Prove in the usual notation that for a branching process {Xn, n=0,1,...} (a) the probability of extension is the smallest positive root of of the equation S=P(s) (b) extinction is certain if and only if the mean number of the off springs per individual does not exceed unity.

21. Explain Yule - Furry process. Find mean and variance.

22. For a renewal process { N(t); t > 0} , find the distribution of N(t).