TM252528H

Reg. No :..............................

Name :..................................

MASTER'S DEGREE (C.S.S) EXAMINATION, MARCH 2025

2020, 2021, 2022, 2023 ADMISSIONS SUPPLEMENTARY

SEMESTER II - CORE COURSE

ST2C08TM - Multivariate Distributions

Time : 3 Hours **Maximum Weight : 30**

Part A

I. Answer any Eight questions. Each question carries 1 weight **(8x1=8)**

1. Let the r. v X has a multinomial distribution with parameter $(n, P_1, P_2, \ldots, P_n)$, obtain the assymptotic distribution of $Y = \sum_{i=1}^{n}[\frac{(x_i - nP_i)^2}{nP_i(1-P_i)}]$. Also obtain the mean and variance of Y.
2. Define first form of Gumbel's bivariate exponential distribution.
3. Show that if (X, Y) has a bivariate normal distribution, X and Y are independent if and only if $\rho = 0$.
4. Derive the distribution of the sample mean vector $\overline{X}$ if $X \sim N_p(\mu, \Sigma)$.
5. Explain characterization property of p variate normal distribution.
6. State and Prove Crames and Wold theorem.
7. Define Wishart distribution.
8. Define generalized variance of the multivariate distribution.
9. Give the necessary and sufficient condition for the independence of two quadratic form.
10. If A is a symmetric matrix X is a random vector, prove that $E(X'AX) = tr(A[E(XX')])$.

Part B

II. Answer any Six questions. Each question carries 2 weight **(6x2=12)**

11. For Gumbel's first form of Bivariate Exponential distribution find the conditional distribution f(y/x).
12. Show that the marginal distribution of y of a bivariate normal distribution is univariate normal with mean μ_2 and variance ${\sigma_2}^2$.
13. If $X \sim N_p(\mu, \Sigma)$, then X can be written as $X = \mu + CY$ where, $\Sigma = CC'$ and $Y \sim N_p(0, I)$.
14. Show that if $X \sim N_p(\mu, \Sigma)$ if and only if every linear combination of X say $T'X \sim N_1(T'\mu, T'\Sigma T)$.
15. Let X and Y be pxp symmetric matrices of real variables and A be a pxp non singular matrix of constants. Then if $Y = AXA'$ show that $dY = |A|^{p+1}\, dX$.
16. Let X be a real symmetric pxp matrix and T is a lower triangular matrix with $t_{jj} > 0, j = 1, 2, \ldots$ then if X = TT' Show that $dX = 2^p \prod_{j=1}^{p}(t_{jj})^{p-j+1} dT$.
17. Explain the estimation of multiple correlation.
18. Let $X_1, X_2, \ldots, X_n$ be n independent random variables such that each $X_i \sim N(0, 1)$. Show that $\sum (x_i - \bar{x})^2$ and $\bar{x}^2$ are independent.

Part C

III. Answer any Two questions. Each question carries 5 weight **(2x5=10)**

19. Derive the Characteristic function of bivariate normal distribution.

20. Show that the mean of a random sample of size N from $N(\mu, \Sigma)$ is distributed according to $N(\mu, \frac{\Sigma}{N})$ and independently of the m.l.e estimate of Σ namely $\frac{A}{N}$.

21. Derive the distribution of the U statistic where, $U = \frac{|A|}{|A+H|}$. Where $A \sim W(\Sigma, n)$, and $H \sim W(\Sigma, m)$. A and H are independently distributed.

22. State and prove Cochran's theorem.