MASTER'S DEGREE (C.S.S) EXAMINATION, MARCH 2025
2020, 2021, 2022, 2023 ADMISSIONS SUPPLEMENTARY
SEMESTER II - CORE COURSE
ST2C06TM - Estimation Theory

Time : 3 Hours   Maximum Weight : 30

Part A

I. Answer any Eight questions. Each question carries 1 weight   (8x1=8)

1. Show that beta distribution belongs to the exponential family.

2. Define Ancillary statistic. State Basu's theorem.

3. Let X_1, X_2 be two i.i.d $P(\lambda)$ random variables. Show that the statistic $T = X_1 + 2X_2$ is not sufficient for λ.

4. Define minimum variance bound estimator. Give the regularity conditions for attainment of it.

5. Define UMVU estimator and give an example.

6. Define confidence interval of estimation.

7. What are shortest confidence interval? Explain how you can find it in the case of symmetric distribution.

8. Define Modified minimum chi square method of estimation.

9. Define Loss function. Explain different loss functions associated with decision rules.

10. Explain Baye's risk.

Part B

II. Answer any Six questions. Each question carries 2 weight   (6x2=12)

11. State and prove sufficient condition for consistency of an estimator.

12. Prove that UMVUE estimator if it exists is unique.

13. Let $f(x; \theta) = \frac{x}{\theta^2} e^{\frac{-x}{\theta}}; x > 0, \theta > 0$. Obtain Cramer Rao lower bound. Find an unbiased estimate of θ obtaining this bound.

14. Show that Minimum Variance Bound Estimator is Unique.

15. Explain Cramer Rao lower bound with an example.

16. Obtain MLE's based on the distributions taken
    (i) *Poisson (λ)*
    (ii) $U(0, \theta)$

17. Differentiate between risk function and loss function

18. Define loss function and mention commonly used loss functions.

Part C

III. Answer any Two questions. Each question carries 5 weight   (2x5=10)

19. State and Prove Rao-Blackwell theroem.

20. Establish C-R inequality hence state and prove necessary and sufficient condition for the attainment of C-R lower bound.

21. Explain the minimum chi square and modified minimum chi square method of estimation. Give example for each method.

22. Let $X_1, X_2, ..., X_n$ be a random sample from $P(\theta)$. For estimating θ assuming squared error loss function, and prior distribution of θ is given by $\pi(\theta) = e^{-\theta}; \theta > 0$ is used. Find the Baye's estimate of θ.