B. Sc. DEGREE (C.B.C.S.S.) EXAMINATION, NOVEMBER 2018

(2014 Admission Supplementary)

SEMESTER V - CORE COURSE (MATHEMATICS)

MAT5DE – DIFFERENTIAL EQUATIONS

Time: Three Hours   Maximum Marks: 80

PART A

I.   Answer all questions. Each question carries 1 mark.

1.   Is the differential equation $y^2 \frac{d^2y}{dx^2} + 3x \frac{dy}{dx} + 5y = e^x$ linear or not?

2.   Solve the equation $\tan y \, dx + x dy = 0$

3.   Find the differential equation of family of curves $y = mx$.

4.   Define linearly independent function.

5.   Find $(D^2 + 2D) \sin 3x$

6.   Write the UC set of the function $x^3 e^{7x}$

7.   Write the general Bessel's equation of order P.

8.   Find the ordinary point of the differential equation
$$x^2 y'' + 2xy' + y = 0$$

9.   Form the partial differential equation by eliminating the arbitrary function.
$$z = f(x + y) + yx$$

10. Write the parametric equation of the sphere.
$$x^2 + y^2 + z^2 = a^2$$

(10x1=10)

PART B

II.   Answer any eight questions. Each question carries 2 marks.

11. Solve $6 x^2 y \, dx - (x^3 + 1)dy = 0$

12. Solve $\frac{dy}{dx} + 3\frac{y}{x} = 6x^2$

13. Solve $(6x + 4y + 1)dx + (4x + 2y + 2)dy = 0$

14. Find the orthogonal trajectories of the family of curves $x^2 + y^2 = c^2$

15. Find a real general solution of $x^2 y'' + 3xy' + y = 0$

16. Solve the equation $y'' - 2y' = 12x$

17. Solve the equation $\frac{d^2y}{dx^2} + 3\frac{dy}{dx} = \cos 2x$

18. Show that $J_{-p}(x) = (-1)^p J_p(x)$ if p is an integer.

19. Find the indicial equation of the $2x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} + (x^2 - 3)y = 0$

20. Discuss the nature of the singularity of the differential equation
$$x^2(x - 2)^2 \frac{d^2y}{dx^2} + 2(x - 2)\frac{dy}{dx} + (x + 1)y = 0$$

21. Derive a partial differential equation by eliminating arbitrary function f and φ from
$$z = f(x + ay) + \varphi(x - ay)$$

22. Show that the direction cosines of the tangent at the point (x,y,z)to the conic

(P.T.O)

$ax^2 + by^2 + cz^2 = 1, x + y + z = 1$ are propositional to $(by - cz, cz - ax, ax - by)$.

<div align="right">(8x2=16)</div>

PART C

III. Answer any six questions. Each question carries 4 marks.

23. Solve $\frac{dy}{dx} + y = xy^3$

24. Solve $\left(x \tan\frac{y}{x} + y\right) dx - x dy = 0$

25. Solve $(x + 2y + 3)dx + (2x + 4y - 1)dy = 0$

26. Find the general solution $x^2 \frac{d^2y}{dx^2} + 4x \frac{dy}{dx} + 2y = 4 \log x$

27. Find the general solution of $\frac{d^2y}{dx^2} + y = 3x + 5 \tan x$.

28. Using the variation of parameters, Solve the equation

$$\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 5y = e^{-2x} \sec x$$

29. Find the power series solution of the initial valued problem

$$(x^2 + 1)\frac{d^2y}{dx^2} + x\frac{dy}{dx} + 2xy = 0, y(0) = 2, y'(0) = 3$$

30. Show that the partial differential equation

$$\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial y^2} = \frac{2z}{x^2}$$

Is satisfied by

$$z = \frac{1}{x}\emptyset(y - x) + \emptyset'(y - x)$$

Where $\emptyset$ is an arbitrary function

31. Find the general solution of the differential equation.

$$x^2\frac{\partial z}{\partial x} + y^2\frac{\partial z}{\partial y} = (x + y)z$$

<div align="right">(6x4=24)</div>

PART D

IV. Answer any two questions. Each question carries 15 marks.

32. Solve $(x - 2y + 1)dx + (4x - 3y - 6)dy = 0$

33. Use the method of Frobenius to find the solution of the differential equation

$$2x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + (x^2 - 3)y = 0$$

34. Solve

a) $\frac{d^2y}{dx^2} + y = 3x + 5 \tan x$

b) $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + 4y = 2x \log x$

35. Find the general integrals of the linear partial differential equations.

a) $z(xp - yq) = y^2 - x^2$

b) $px(x + y) = qy(x + y) - (x - y)(2x + 2y + z)$

<div align="right">(2x15=30)</div>