TM252995D

Reg. No :...............................

Name :...................................

**MASTER'S DEGREE (C.S.S) EXAMINATION, MARCH 2025**

**2020, 2021, 2022, 2023 ADMISSIONS SUPPLEMENTARY**

**SEMESTER II - CORE COURSE MATHEMATICS**

**MT2C06TM20 - Abstract Algebra**

**Time : 3 Hours** **Maximum Weight : 30**

## Part A

**I. Answer any Eight questions. Each question carries 1 weight** **(8x1=8)**

1. (i) Prove or disprove : Every abelian gorup of order $p^2$ is cyclic
   (ii) Find the maximum possible order of some element in $Z_8 \times Z_{10} \times Z_{24}$
2. Prove that, Let X be a G-Set then prove that $G_x$ is a subgroup of G for each x in X.
3. Define with examples (i) Decomposable Group (ii) Indecomposable Group.
4. State and prove 3rd Isomorphism theorem. Illustrate the theorem with the help of an example.
5. Find all the Sylow-p-subgroups of S3 where p = 2, 3.
6. (a)Let H and N be two subgroups of G, Define (i) HN (ii) join of H and N (b) Define Solvable group
7. State and prove Euler's Generalization of Fermat's Theorem
8. i. Define irreducible polynomial and reducible polynomial and give examples.
9. Show that an isomorphism of a ring R with a ring R' is a homomorphism $\emptyset: R \to R'$ with $Ker(\phi) = \{0\}$
10. Prove that the set End(A) of all endomorphism of an abelian group A forms a ring under homomorphism addition and homomorphism multiplication (function composition).

## Part B

**II. Answer any Six questions. Each question carries 2 weight** **(6x2=12)**

11. i. Classify the group $(Z_4 \times Z_2)/\{0\} \times Z_2$ according to the fundamental theorem of finitely generated abelian groups.
    ii. Prove that, the finite indecomposable abelian groups are exactly the cyclic groups with order a power of a prime.
12. (i) How many distinguishable ways can seven people be seated at a round table, where there is no distinguishable head to the table?
    (ii) How many disntinguishable necklaces (with no clasp) can be made using seven different colored beads of the same size?
13. State and prove Third Sylow Theorem
14. Let N is a normal subgroup of G and if H is any subgroup of G, then H$\vee$N=HN=NH. Furthermore, If H is also normal in G, then prove that HN is normal in G.
15. Prove that, If G is a finite subgroup of the multiplicative group $< F^*, . >$ of a field F, then G is cyclic. In particular, the multiplicative group of all nonzero elements of a finite field is cyclic
16. Let p be a prime ≥ 3. Use Wilson's theorem to find the remainder of (p-2)! Modulo p.

17. (i) If R is a ring with unity , then show that the map $\varphi : \mathbb{Z} \to R$ given by $\varphi(n) = n \cdot 1$ for $n \in \mathbb{Z}$ is a homomorphism

(ii) Define ideal of a ring

18. (i) Let $G = \{e, a\}$ be a cyclic group of order 2 and $Z_2 = \{0, 1\}$ is a field. Find the group Algebra $Z_2G$

(ii) Define ring homomorphism and its kernel.

## Part C

**III. Answer any Two questions. Each question carries 5 weight** **(2x5=10)**

19.

i. State and prove Burnside's Formula

ii. If G is a finite group and X is a finite G-set, then prove that the number of orbits in X under G = $\frac{1}{|G|}\sum_{g\in G} |X_g|$

20. State and prove the 2nd and 3rd Isomorphism theorem.

21. If an integral domain D is given, construct a field of quotients F such that D can be embedded in F.

22. (i) Prove that a commutative ring with unity is a field iff it has no proper non trivial ideals. (ii) Prove or Disprove, The maximal ideals of Z are precisely the ideals pZ for prime positive integers p.