TM252556G

Reg. No :...............................

Name :..................................

MASTER'S DEGREE (C.S.S) EXAMINATION, MARCH 2025

2020, 2021, 2022, 2023 ADMISSIONS SUPPLEMENTARY

SEMESTER II - CORE COURSE

MT2C08TM20 - Advanced Complex Analysis

Time : 3 Hours **Maximum Weight : 30**

Part A

I. Answer any Eight questions. Each question carries 1 weight **(8x1=8)**

1. Define Poisson integral of a piecewise continuous function U and show that the Poisson integral of a constant is a constant.
2. Write a brief note on Dirichlet's problem.
3. Derive the Maclaurin series for $\sin^{-1} z$.
4. Show that $\prod_{n=2}^{\infty} (1 - \frac{1}{n^2}) = \frac{1}{2}$.
5. Prove: (a) $\Gamma(1) = 1$, (b) $\Gamma(n) = (n-1)!$
6. Show that $\zeta(s)\,\Gamma(s) = \int_0^{\infty} \frac{x^{s-1}}{e^x - 1}\,dx$ where $\sigma > 1$.
7. Briefly describe the zeros of Zeta function.
8. State and prove a necessary and sufficient condition for a family of functions to be normal.
9. Define primitive period of a function. Give an example.
10. Explain the fundamental region of the unimodular group.

Part B

II. Answer any Six questions. Each question carries 2 weight **(6x2=12)**

11. Derive Schwarz's formula.
12. Define a subharmonic function and show that every harmonic function is trivially subharmonic. Give an example of a subharmonic function which is not harmonic.
13. Find the Taylor series expansion of $\frac{1}{z}$ at z = 1 and z = 2i.
14. Express $\sin \pi z$ in the form of canonical product.
15. Let f be a topological mapping of a region Ω onto a region Ω'. If $\{z_n\}$ or z(t) tends to the boundary of Ω, then show that $\{f(z_n)\}$ or f (z(t)) tends to the boundary of Ω'.
16. Prove that the function $\varphi(s) = \frac{1}{2} s(1-s)\pi^{-s/2}\,\Gamma\left(\frac{s}{2}\right)\zeta(s)$ is entire and satisfies $\varphi(s) = \varphi(1-s)$.
17. Drive the relation connecting Weierstrass Zeta function and sigma function.
18. Prove that any two bases of the same module are connected by a unimodular transformation.

Part C

III. Answer any Two questions. Each question carries 5 weight **(2x5=10)**

19. (a) Derive the Gauss Mean Value property for harmonic functions.

(b) Prove that the function $P_U(z)$ is harmonic for $|z| < 1$ and show that $\lim_{z \to e^{i\theta_0}} P_U(z) = U(\theta_0)$ provided that U is continuous at θ_0.

20. (a) Find Mittag-Leffler representation of $\pi \cot \pi z$.

    (b) Derive Legender's Duplication Formula.

21. Prove the functional equation $\zeta(s) = 2^s \pi^{s-1} \sin\left(\frac{\pi s}{2}\right) \Gamma(1-s)\zeta(1-s)$.

22. Establish the theorem on the existence of the canonical basis for the discrete module M of a meromorphic function.