TM252623X

Reg. No :..............................

Name :..............................

MASTER'S DEGREE (C.S.S) EXAMINATION, MARCH 2025
2020, 2021, 2022, 2023 ADMISSIONS SUPPLEMENTARY
SEMESTER II - CORE COURSE MATHEMATICS
MT2C07TM20 - Advanced Topology

Time : 3 Hours **Maximum Weight : 30**

Part A

I. Answer any Eight questions. Each question carries 1 weight **(8x1=8)**

1. Prove that a compact space in a Hausdorff space is closed.
2. (a) Define (i) Point wise Convergence. (ii) Uniform Convergence. (b) Prove or Disprove Point wise Convergence implies Uniform Convergence.
3. Define standard sub-base and standard base of a product topology
4. Prove that if each co-ordinate space is T_0 then the topological product has the corresponding property.
5. Define (i) Cartesian product of indexed family of sets. (ii) i-th co-ordinate of x, where x is a function on the set S = {1, 2, 3, ..., n}. (iii) i-th factor of the Cartesian product.
6. Define distinguish points
7.

   i. Prove $[0,1]^T$ is Tychnoff

   ii. Show that every Hilbert cube is metrisable
8. Prove that, Every continuous real valued function on a Countably compact space is bounded and attains its extrema
9. Define with examples (i) Direct set (ii) Net in X.
10. Define (i) Straight line homotopy (ii) Positive linear map

Part B

II. Answer any Six questions. Each question carries 2 weight **(6x2=12)**

11. State and prove Uniform Convergence theorem.
12. Prove that, "Every map from a compact space into a T2-space is closed. The range of such a map is a quotient space of the domain."
13. Prove that if each co-ordinate space is regular then the topological product has the corresponding property.
14. Show that being Hausdroff is a productive property
15. Prove that, Suppose $\{Y_i/\in I\}$ is an indexed family of sets, Z is a set and $\{\theta_i : Z \to Y_i / i \in I\}$ is a family of functions such that for any set X and any family $\{f_i : X \to Y_i / i \in I\}$ of functions, there exists a unique function $e: X \to Z$ satisfying $\theta_i \circ e = f_i, \forall i \in I$. Then there exists a bijection h from Z to $\prod Y_i$ such that for each $i \in I, \theta_i = \pi_i \circ h$. Moreover this bijection is unique. In other words, up to a bijection Z is the product of $Y_i' s \; and \; \theta_i 's$ are the projection functions.
16. Prove that the Let $\{f_i : X \to Y_i / i \in I\}$ be a family of functions which distinguishes points from closed sets in X then the corresponding evaluation function $e: X \to \prod_{i \in I} Y_i$ is open when regarded as a function from X

to e(X).

17. Suppose $S: D \to X$ is a net and F is a cofinal subset of S. If $S/F: F \to X$ converges to a point $x \in X$ then prove that x is a cluster point of S.

18. Show that if h, h': X → Y are homotopic and k, k': Y → Z are homotopic, then k ∘ h and k '∘ h' are homotopic.

Part C

III. Answer any Two questions. Each question carries 5 weight **(2x5=10)**

19. Show that, if A is a closed subset of a normal space X and suppose $f: A \to [-1.1]$ is a continous function. Then there exist a continous function $F: X \to [-1.1]$ such that . Discuss the same if [-1,1] replaced with (-1,1).

20. State and prove the necessary and sufficient condition for the product space is connected.

21. Show that a topological space is embeddable in the Hilbert cube if it is second countable and T3. Discuss it's converse.

22. Given spaces X and Y , let [X, Y] denote the set of homotopy classes of maps of X into Y . (i) Let I = [0, 1]. Show that for any X, the set [X, I] has a single element. (ii) Show that if Y is path connected, the set [I, Y] has a single element.

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