TB145620D

Name........................

Reg. No......................

B. Sc. DEGREE (C.B.C.S.S) EXAMINATION, NOVEMBER 2018
(2014 Admission Supplementary)
SEMESTER V - CORE COURSE (MATHEMATICS)
MAT5AA - ABSTRACT ALGEBRA

Time: Three Hours **Maximum Marks: 80**

PART A

I. Answer all questions. Each question carries 1 mark.

1. How many different binary operations can be defined on a set having exactly 2 elements
2. Give an example of an abelian group which is not cyclic
3. Find the order of the permutation $\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 1 & 5 & 4 & 6 & 3 \end{pmatrix}$ in S_6
4. Is $\langle Z,+\rangle$ isomorphic to $\langle Q,+\rangle$
5. Define normal subgroup of a group G.
6. What are the units of Z_5
7. Define Kernel of a group homomorphism
8. Define Skew field.
9. Check whether Q is an ideal of R.
10. Find the characteristic of the ring Z_n. **(10x1=10)**

PART B

II. Answer any eight questions. Each question carries 2 marks.

11. Show that every group G with identity e and such that $x * x = e$ for al $x \in G$ isabelian.
12. Prove left cancellation law.
13. State that the set G={1,- 1, i, -i} forms an abelian group with respect to multiplication
14. Prove that every cyclic group is abelian
15. Show that R under addition is isomorphic to R^+ under multiplication
16. Prove that every subgroup of an abelian group is a normal subgroup
17. If $\phi : G \to G'$ is a group homomorphism, then prove that Ker ϕ is a normal subgroup of G.
18. Find all solutions of the equation $x^2+2x+4=0$ in Z_6.
19. Let $\phi : G \to G'$ be a homomorphism, then prove that if $a \in G$ then $a^{-1}\phi = (a\phi)^{-1}$.
20. Check whether $\langle Z,+,\cdot\rangle$ is a field. Give reasons to support of your answer.
21. Let R be a commutative ring with unity of characteristic 4. Compute and simplify $(a+b)^4$ for $a, b \in R$.
22. Find all ideals of Z_{12}.

(8x2= 16)

PART C

III. Answer any six questions. Each question carries 4 marks.

23. Show that if H and K are subgroups of an abelian group G then $G' = \{hk / h \in H \text{ and } k \in K\}$ is a subgroup of G
24. Let G be a group and $a \in G$. Then $H = \{a^n / n \in Z\}$ is a subgroup of G and is the smallest

(P.T.O)

subgroup G that contains a.

25. Let G be a cyclic group with n elements and generated by a. Let $b \in G$ and $b = a^s$. then b generates a cyclic subgroup H of G containing $\frac{n}{d}$ elements, where d is the g.c.d of n and s.

26. a). Find all generators of Z_8.
    b).Find all subgroups of Z_8 and draw the lattice diagram.
27. Prove that any infinite cyclic group G is isomorphic to the group Z of integers under addition.
28. Prove that a homomorphism ϕ of a group G is a one-one function iff Ker $\phi=\{e\}$.
29. Prove that every finite integral domain is a field.
30. A).Define characteristic of a ring.
    B).Prove that if R is a ring with unity 1, then R has characteristic n >0 iff n is the smallest positive integer such that n.1 = 0.
31. Let R be a commutative ring and $a \in R$. Show that $I_a=\{x \in R/ax = 0\}$ is an ideal of R.

(6x4= 24)

PART D

IV. Answer any two questions. Each question carries 15 marks.

32. a).Prove that for $n \geq 2$, the number of even permutations in S_n is same as the number of odd permutations.
    b).Prove that intersection of two subgroups of a group G is a subgroup of G.
33. State and prove Cayley's theorem.
34. State and prove Fundamental Homomorphism theorem for groups.
35. a).Prove that if R is a ring with unity and N is an ideal of R containing a unit then N = R.
    b).Prove that M is a maximal normal subgroup of G iff G/M is simple.

(2x15=30)