TB245765F

6/6/19.11

Reg. No :...............................

Name :.................................

BACHELOR'S DEGREE (C.B.C.S) EXAMINATION, NOVEMBER 2024

2022 ADMISSIONS REGULAR

SEMESTER V - CORE COURSE (MATHEMATICS)

MT5B06B18 - Real Analysis-I

Time : 3 Hours **Maximum Marks : 80**

Part A

I. Answer any Ten questions. Each question carries 2 marks **(10x2=20)**

1. State order completeness property in R
2. List four lower bounds of the set of Prime numbers.
3. Define countable set and give an example.
4. Prove that superset of a neighborhood of a point x is also a neighborhood of x.
5. Find all adherent points of the set {4, 6} ?
6. Find the interior of the set of all negative integers?
7. Define monotonically decreasing sequences. Give an example.
8. Define an "oscillate infinitely sequence" . Give an example.
9. Find $\lim_{n\to\infty} \frac{3+2\sqrt{n+1}}{\sqrt{n+1}}$.
10. Show that for any real number x , $\lim \frac{x^n}{n!}$ = 0 .
11. Define a Cauchy sequence in a metric space (X, d).
12. Identify two disjoint subsets M and N of the metric space (R, d) where R is the set of real numbers and d is the usual metric such that d (M, N) = 0

Part B

II. Answer any Six questions. Each question carries 5 marks **(6x5=30)**

13. State and prove Archimedean property of real numbers.
14. For any h >0, prove that there exist a positive integer n such that 1/n < h.
15. Prove that closure of a bounded set is bounded.
16. Prove or disprove : The intersection of a finite family of open sets is an open set.
17. Prove that every bounded sequence with unique limit point is convergent.
18. Explain briefly about non - convergent sequences with suitable examples.
19. Using Cauchys general principle of convergence show that the sequence { s_n } where s_n = 1 + 1/2 + 1/3 + ... 1/n is not convergent .
20. In any metric space (X, d), determine whether the union of an arbitrary family of open sets is open.
21. Let A and B be two non empty subsets of a metric space (X, d). Then prove or disprove : "d(A, B) = 0 if and only if $A \cap B = \phi$"

Part C

III. Answer any Two questions. Each question carries 15 marks **(2x15=30)**

22. Establish the equivalence of Dedekind's form of completeness property and Order completeness property of real numbers.

23. Prove that: (i) The derived set of a bounded set is bounded. (ii) The derived set S' of a bounded infinite set $S \subseteq R$ has the smallest and the greatest members.

24. Prove that a necessary and sufficient condition for the convergence of a monotonic sequence is that it is bounded .

25. Let (X, d) be a complete metric space and Y be a subspace of X. Then prove that Y is complete if and only if it is closed in (X, d).