

B. Sc. DEGREE (C.B.C.S.S.) EXAMINATION, NOVEMBER 2024
(2016 & 2017 Admission Supplementary)
SEMESTER V- CORE COURSE (MATHEMATICS)
MT5B08B - GRAPH THEORY

Time: Three Hours

Maximum Marks: 80

PART A**I. Answer all the questions. Each question carries 1mark.**

1. State True or False: "Every trail is a path."
2. Draw the complement of K_3
3. Give an example of a tree with 5 edges.
4. Give an example of a complete bipartite graph.
5. Give an example of a perfect matching.
6. Define a tournament.

(6x1 = 6)**PART B****II. Answer any seven questions. Each question carries 2 marks.**

7. Draw two spanning subgraphs of K_4
8. Find the radius and diameter of the Petersen graph.
9. Define incidence matrix of a graph. What is the sum of the elements of its i^{th} row?
10. Let G be a connected graph. Then prove that G is a tree if and only if for every edge of G is a bridge.
11. State the first theorem of Digraph theory.
12. Define closure of a simple graph.
13. State Redei's theorem.
14. Define cut vertex of a graph with an example.
15. Define Hamiltonian graph with an example.
16. Define an Euler digraph.

(7x2 = 14)**PART C****III. Answer any five questions. Each question carries 6 marks.**

17. For any two vertices u and v of a graph G , prove that every u - v walk contains a u - v path.
18. Explain Chinese Postman Problem.
19. Define closure of a graph and prove that a simple graph G is Hamiltonian if and only if its closure is Hamiltonian.

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20. Let D be a weakly connected digraph with at least two vertices. If D has two vertices u and v such that $\text{od}(u) = \text{id}(u) + 1$ and $\text{id}(v) = \text{od}(v) + 1$ and, for all other vertices w of D , $\text{od}(w) = \text{id}(w)$ then prove that D has a directed Euler trail
21. Explain the Personnel Assignment problem.
22. Explain Konisberg bridge problem .
23. Draw the four tournaments on 4 vertices.
24. Define orientation of a graph and draw all the possible orientations of K_3 . How many of them are strongly connected?

(5x6=30)

PART D

IV. Answer any two questions. Each question carries 15 marks.

25. Prove that a non- empty graph G with at least two vertices is bipartite if and only if it has no odd cycles.
26. State and prove Whitney's theorem.
27. a) Explain Marriage problem.
b) Let G be a k -regular bipartite graph with $k > 0$. Then prove that G has a perfect matching.
28. Show that a weakly connected digraph D with at least one arc is Euler if and only if $\text{od}(v) = \text{id}(v)$ for every vertex ' v ' of D .

(2x15=30)

