TB245967N

Reg. No :...............................

Name :..................................

BACHELOR'S DEGREE (C.B.C.S) EXAMINATION, NOVEMBER 2024

2022 ADMISSIONS REGULAR

SEMESTER V - CORE COURSE (MATHEMATICS)

MT5B08B18 - Abstract Algebra

Time : 3 Hours

Maximum Marks : 80

Part A

I. Answer any Ten questions. Each question carries 2 marks **(10x2=20)**

1. Give an example of an abelian group which is not cyclic.
2. Calculate the number of different commutative binary operations that can be defined on a set S of two elements ?
3. Define transposition.
4. Define the orbits of a permutation.
5. List the generators of a cyclic group having order 16.
6. Compute the gcd of 58 and 68.
7. Define normal subgroup.
8. Define inner automorphism of a group G.
9. Find the index of $< 3 >$ in the group $\mathbb{Z}_{24}$
10. Let $\mu = (1, 2, 4, 5)(3, 6)$ in S_6. Find the index of $< \mu >$ in S_6.
11. Define the characteristic of a ring.
12. Determine whether the matrix ring $M_2(\mathbb{Z}_2)$ have divisors of zero?

Part B

II. Answer any Six questions. Each question carries 5 marks **(6x5=30)**

13. Show that the inverse of each element is unique in a group.
14. Let G be the multiplicative group of all invertible $n \times n$ matrices with entries in $\mathbb{C}$ and let T be the subset of G consisting of those matrices with determinant 1. Show that T is a subgroup of G.
15. Prove or disprove: A group is cyclic if and only if it is abelian.
16. Let A be a nonempty set and let S_A be the collection of all permutations of A. Deduce that S_A is a group under permutation multiplication.
17. Find all subgroups of $\mathbb{Z}_{12}$ and draw the lattice diagram
18. Let ϕ be a homomorphism of a group G into a group G'. If K' is a subgroup of $G' \cap \phi[G]$, then prove that $\phi^{-1}[K']$ is a subgroup of G.
19. Find all cosets of the subgroup $< 18 >$ of $\mathbb{Z}_{36}$
20. Define ideal. Show that $n\mathbb{Z}$ is an ideal of $\mathbb{Z}$.
21. If R is a ring with additive identity 0, then deduce that
    1. $a(-b) = (-a)b = -(ab)$
    2. $(-a)(-b) = ab$

Part C

III. Answer any Two questions. Each question carries 15 marks **(2x15=30)**

22. Let F be the set of all real-valued functions with domain $\mathbb{R}$ and let $\tilde{F}$ be the subset of F consisting of those functions that have a nonzero value at every point in $\mathbb{R}$. Determine whether the given subset of F with the induced operation is a subgroup of the group F under addition.

a) The subset of all $f \in \tilde{F}$ such that $f(1) = 0$

b) The subset of all $f \in F$ such that $f(1) = 0$

c) The subset $\tilde{F}$

23. a)Explain permutation and its orbits. b)Explain identity permutation. Compute the orbits of the identity permutation on 5 letters. c) Illustrate the definition of cycle with an example. Is multiplication of cycles commutative? Justify your answer.

24. If $n \geq 2$, establish that the collection of all even permutations of $\{1, 2, 3, \ldots, n\}$ forms a subgroup of order $\frac{n!}{2}$ of the symmetric group S_n.

25. Establish that M is a maximal normal subgroup of a group G if and only if G/M is simple.