

## BACHELOR'S DEGREE (C.B.C.S) EXAMINATION, NOVEMBER 2024

## 2022 ADMISSIONS REGULAR

## SEMESTER V - CORE COURSE (MATHEMATICS )

## MT5B07B18 - Differential Equations

Time : 3 Hours

Maximum Marks : 80

## Part A

I. Answer any Ten questions. Each question carries 2 marks

(10x2=20)

1. Compute the wronskian and show that  $e^x, e^{2x}$  and  $e^{3x}$ . Determine whether they are linearly independent.
2. Solve  $(y^2 + 3) dx + (2xy - 4) dy = 0$ .
3. Solve  $\tan \theta dr + 2r d\theta = 0$ .
4. Solve the auxiliary equation of  $4 \frac{d^2 y}{dx^2} - 12 \frac{dy}{dx} + 5y = 0$ .
5. Evaluate the Wronskian of  $\{x, 2x, 3x^2+3\}$  and check whether they are linearly independent.
6. Solve  $4 \frac{d^2 y}{dx^2} + y = 0$ .
7. Write the linear combination of functions with undetermined coefficients to form the particular integral of the given differential equation  $\frac{d^2 y}{dx^2} + 9y = e^{3x} + \sin 3x$  without actually solving it.
8. State the two conditions to check the regularity of singular points
9. Give an approximate value of Euler's constant
10. Write the Bessel's function of the first kind of order 0.
11. Write the partial differential equation  $z = x + y + f(xy)$ .
12. Write the partial differential equation  $z = ax + a^2 y^2 + b$

## Part B

II. Answer any Six questions. Each question carries 5 marks

(6x5=30)

13. Solve  $(2x^2 y + y) dx + (2y^3 - x) dy = 0$
14. Solve  $x \sin y dx + (x^2 + 1) \cos y dy = 0$ .
15. Solve  $\frac{d^2 y}{dx^2} - 4 \frac{dy}{dx} + 3y = 9x^4 + 4$ .
16. Given that  $y = x$  is a solution of  $x^2 \frac{d^2 y}{dx^2} - 4x \frac{dy}{dx} + 4y = 0$ . Evaluate a linearly independent solution by reducing the order.
17. Solve the system  $2 \frac{dx}{dt} - 2 \frac{dy}{dt} - 3x = t$ ,  
 $2 \frac{dx}{dt} + 2 \frac{dy}{dt} + 3x + 8y = 2$

18. Solve  $\frac{dx}{dt} + \frac{dy}{dt} + 2y = \sin t$ ,  
 $\frac{dx}{dt} + \frac{dy}{dt} - x - y = 0$ .

19. Compute the power series solution of  $\frac{d^2y}{dx^2} + x \frac{dy}{dx} + y = 0$ .

20. Solve  $(y+2)p - (x+2)q = x - y$

21. Solve the partial differential equation  $\frac{dx}{x(z^2 - y^2)} = \frac{dy}{y(x^2 - z^2)} = \frac{dz}{z(y^2 - x^2)}$ .

### Part C

III. Answer any Two questions. Each question carries 15 marks

(2x15=30)

22. Solve  $(6x+4y+1)dx + (4x+2y+2)dy = 0$ .

23. a) Given that  $y = x^2$  and  $y = x^5$  are linearly independent solutions of the corresponding homogenous equation of  $x^2 \frac{d^2y}{dx^2} - 6x \frac{dy}{dx} + 10y = 3x^4 + 6x^3$ . Compute the general solution.

b) Solve  $\frac{d^2y}{dx^2} + 2 \frac{dy}{dx} + 2y = 10 \sin 4x$ .

24. Use Frobenius method to solve  $x^2 \frac{d^2y}{dx^2} + (x^2 - 3x) \frac{dy}{dx} + 3y = 0$ .

25. Solve a)  $x^2(y-z)p + y^2(z-x)q = z^2(x-y)$   
 b)  $(x^2 - y^2 - z^2)p + 2xyq = 2xz$ .