

BACHELOR'S DEGREE (C.B.C.S) EXAMINATION, NOVEMBER 2024

2023 ADMISSIONS REGULAR

SEMESTER III - CORE COURSE (STATISTICS)

ST3C03B23 - Probability Distributions

Time : 3 Hours

Maximum Marks : 80

Part A

I. Answer any Ten questions. Each question carries 2 marks

(10x2=20)

1. Define mathematical expectation.
2. Find the m.g.f of $Y=2X-5$
3. Define moment generating function of a random variable.
4. Express mean of a random variable in terms of its m.g.f.
5. Define Poisson Distribution. Also obtain its moment generating function.
6. Show that for the Geometric distribution $P(x+1) = qP(x)$
7. If X and Y are independent Poisson variates with means 2 and 3 respectively find the mean and variance of $2X+3Y$.
8. State and prove the additive property of the gamma distribution.
9. If X follows Uniform distribution over $[0,1]$, then state the distribution $Y = -2 \log X$.
10. Obtain the mgf of a gamma distribution with parameter p .
11. State the assumptions in the Lindberg-Levy form of Central limit theorem.
12. Define convergence in probability

Part B

II. Answer any Six questions. Each question carries 5 marks

(6x5=30)

13. Define characteristic function. Prove any two properties?
14. Find the moment generating function of a random variable X whose p.d.f is $f(x) = a^x b$; $x = 0, 1, 2, \dots$, where $a+b=1$. Hence find $V(X)$.
15. For a random variable X , we have $2\log M_x(t) = 30t + 90t^2$. Find its mean, variance and third central moment.
16. If X and Y be two independent random variables each representing the number of failures preceding the first success in a sequence of bernoulli trials with p as probability of success in a single trial and q probability of failure, show that $P(x = y) = \frac{p}{1+q}$
17. Define a Bernoulli random variable. Show that sum of independent Bernoulli random variables follow Binomial distribution.
18. Show that Beta distribution of the first type can be obtained from Beta distribution of the second type by means of a transformation.
19. Find the mean and variance of the Beta distribution of the first type.
20. A distribution with unknown mean μ and has variance equal to 1.5. By using central limit theorem find how large a sample should be taken in order that the probability will be atleast 0.95 that the sample mean will be within 0.5 of the population mean.
21. State central limit theorem and its assumptions

Part C

III. Answer any Two questions. Each question carries 15 marks

(2x15=30)

22. The following table presents the bivariate distribution of a pair of random variables (X,Y). Calculate $E(X)$, $E(Y)$, correlation between X and Y and also $V(X/Y)$.

		X		
Y		0	1	2
	0	.01	.01	.02
	1	.22	.10	.43
	2	.07	.08	.06

23. 1) Fit a Poisson Distribution to the following data and calculate the theoretical frequencies.

X	0	1	2	3	4
F	122	60	15	2	1

- 2) For the Poisson distribution with mean m , find β_1 and β_2 .
24. a) Define Normal Distribution. b) State the conditions at which Binomial Distribution tends to Normal Distribution.
c) Show that binomial distribution tends to Normal Distribution.
25. A random sample of size 100 is taken from an infinite population with mean 75 and variance 256
- (a) Using Tchebychev's inequality, find $P[67 < \bar{X} < 83]$
- (b) Using Central limit theorem, find $P[67 < \bar{X} < 83]$