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Reg. No :..............................

Name :..............................

BACHELOR'S DEGREE (C.B.C.S) EXAMINATION, NOVEMBER 2024

2018, 2019, 2020, 2021, 2022 ADMISSIONS SUPPLEMENTARY

SEMESTER III - COMPLEMENTARY COURSE (STATISTICS) (MATHS & PHY)

ST3C01B18 - Probability Distributions

Time : 3 Hours

Maximum Marks : 80

Part A

I. Answer any Ten questions. Each question carries 2 marks **(10x2=20)**

1. A random variable X has p.d.f. $f(x) = 2^{-x}$; x = 1,2,3, . . ., find mode of the distribution.
2. Write any two limitations of mgf.
3. For a discrete random variable X, show that E(aX + b) = aE(X) + b.
4. Write any two properties of expectation of a random variable.
5. If X ~ P(λ), find E(X).
6. Compute the mode of B(7, ¼).
7. If X_1 and X_2 are two independent random variables following Bernoulli distribution with parameter p, then show that $X_1 + X_2$ follows binomial distribution B(2, p)
8. State and prove the lack of memory property of the exponential distribution.
9. Compute the m.g.f. of Uniform distribution over (0, 2).
10. Obtain the points of inflexion of the normal curve $N(\mu, \sigma)$.
11. State the Lindberg-Levy form of Central limit theorem.
12. State the assumptions in the Lindberg-Levy form of Central limit theorem.

Part B

II. Answer any Six questions. Each question carries 5 marks **(6x5=30)**

13. Distinguish between raw moments and central moments. Obtain the general expression for the r-th central moment in terms of raw moments.
14. Find E(X), if $f(x) = \frac{e^{-1}}{x!}$, x = 0,1,2,3,..... is the p.d.f of X.
15. Find the moment generating function of a random variable X whose p.d.f is $f(x) = a^x b$; x = 0,1,2,. . . . ., where a+b=1. Hence find V(X).
16. Let X and Y be independent random variables such that $P(X= r) = P(Y= r) = q^r p$, r = 0, 1, 2,. .... where p and q are positive numbers such that p + q = 1. Find (1) the distribution of X+Y (2) the conditional distribution of X given X+Y = 3.
17. Obtain the mgf of the binomial distribution and deduce its additive property of independent binomial variates.
18. If X is a random variable distributed as N(0,1) then show that X^2 has gamma distribution with parameters m=1/2 and p= 1/2
19. For a rectangular distribution f(x) = k, $1 \le x \le 2$, show that A.M > G.M > H.M
20. A random sample of size n was taken from a population with mean μ and Standard deviation σ. Find the distribution of the sample mean $\bar{X}$ for large n.

21. How many trials should be performed so that the probability of obtaining at least 40 successes is at least 0.95, if the trials are independent and probability of success in a single trial is 0.2?

Part C

III. Answer any Two questions. Each question carries 15 marks **(2x15=30)**

22. (a)Define conditional expectation and conditional variance.(b) If f(x,y)= x+y; 0<x<1, 0<y<1 is the joint p.d.f. of (X,Y), calculate correlation between X and Y.

23. (1) For the Poisson distribution with mean m show that $\beta_1 = \frac{1}{m}$ and $\beta_2 = 3 + \frac{1}{m}$.

(2) Fit a Poisson distribution to the following data and calculate the theoretical frequencies.

x	0	1	2	3	4
f	122	60	15	2	1

24. (a) Derive the expression for the even ordered central moments of the normal distribution N(μ, σ). (b) find the probability that the number of heads lie in the range 185 and 220 when a fair coin is tossed 400 times.

25. For the random variable X following geometric distribution $f(x) = \frac{1}{2^x}$, x = 1, 2, 3,.... Show that $P(|x-2| \leq 2) = \frac{15}{16}$. Find a lower bound of this probability using Tchebychev's inequality.