TB243273G

Reg. No :.....

Name :.....

BACHELOR'S DEGREE (C.B.C.S) EXAMINATION, NOVEMBER 2024

2023 ADMISSIONS REGULAR

SEMESTER III - CORE COURSE MATHEMATICS

MT3C03B23 - Calculus

Time : 3 Hours **Maximum Marks : 80**

Part A

I. Answer any Ten questions. Each question carries 2 marks **(10x2=20)**

1. If $y = \frac{logx}{x}$, find $\frac{d^2y}{dx^2}$.

2. If $y = Acos2x + Bsin2x$, then prove that $\frac{d^2y}{dx^2} + 4y = 0$.

3. Find the nth derivative of $e^{3x}sin(x+2)$.

4. Evaluate $\frac{\partial w}{\partial u}$ where $w = sin(2x-y), x = u + sinv, y = uv$.

5. Find $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$, where $f(x,y) = \frac{x}{x^2+y^2}$.

6. Evaluate $\frac{dw}{dt}$, where $w = xy + z, x = cost, y = sint, z = t$.

7. Compute the length of the curve $y = \int_{-2}^{x} \sqrt{3t^4 - 1}dt, -2 \leqslant x \leqslant -1$.

8. Evaluate $\int_0^3 \sqrt{y+1}dy$

9. Evaluate $\int_0^{\frac{\pi}{4}} tanx.sec^2xdx$.

10. State Domination Rule for Double Integrals.

11. Evaluate $\int_0^1 \int_1^2 (x^2+y^2)dxdy$

12. Compute $\int_0^2 \int_{-\pi}^{0} \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} (\rho^3 sin2\phi)d\phi d\theta d\rho$.

Part B

II. Answer any Six questions. Each question carries 5 marks **(6x5=30)**

13. If $y = e^{-x}(Ax+B)$, prove that $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = 0$.

14. Expand $2x^3 + 7x^2 + x - 6$ in powers of (x-2).

15. If $y = a(1-cost), x = a(t+sint)$, find $\frac{d^2y}{dx^2}$.

16. Show that $w = sin(x + ct)$ satisfies the equation $\frac{\partial^2 w}{\partial t^2} = c^2 \frac{\partial^2 w}{\partial x^2}$.

17. if $u = sin(xy)$, prove that $u = sin(xy)$, prove that $x\frac{\partial u}{\partial x} - y\frac{\partial u}{\partial y} = 0$

18. The region bounded by the curve $y = \sqrt{x}$, the x-axis and the line $x = 4$ is revolved about the y axis to generate a solid. Compute the volume of the solid

19. Calculate the volume of the solid generated by revolving the region bounded by $y = \sqrt{x}$ and the lines $y = 1, x = 4$ about the line $y = 1$

20. Compute the area of the region that lies inside the cardioid $r = 1 + cos\theta$ and outside the circle $r = 1$.

21. Sketch the region of integration for the integral $\int_0^1 \int_y^{\sqrt{y}} dxdy$ and write an equivalent double integral with order of integration reversed. Also evaluate the integral.

Part C

III. Answer any Two questions. Each question carries 15 marks **(2x15=30)**

22. a)If $y = cos(msin^{-1}x)$, prove that $\left(1 - x^2\right) y_{n+2} - (2n + 1) y_{n+1} + \left(m^2 - n^2\right) y_n = 0$ and then find $y_n(0)$, the nth derivative of y at x=0.

b) If $I_n = \frac{d^n}{dx^n}(x^n logx)$, then prove that $I_n = nI_{n-1} + (n - 1)!$

23. a) Find all local maxima, local minima and saddle point of $f(x, y) = x^2 + xy + y^2 + 3x - 3y + 4$.

b) Find the shortest distance from the origin to the hyperbola $x^2 + 8xy + 7y^2 = 225$.

24. (a) Calculate the area of the region enclosed by the curve $y = 2x - x^2$ and the line $y = -3$

(b) Compute the area of the surface generated by revolving the curve $y = x^3, 0 \leq x \leq \frac{1}{2}$ about the x-axis.

25. (a) Determine the volume of the region D enclosed by the surfaces $z = x^2 + 3y^2$ and $z = 8 - x^2 - y^2$.

(b) Writw the spherical coordinate equation for the cone $z = \sqrt{x^2 + y^2}$.