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TB243562X

Reg. No :...............................

Name :..................................

BACHELOR'S DEGREE (C.B.C.S) EXAMINATION, NOVEMBER 2024

2023 ADMISSIONS REGULAR

SEMESTER III - COMPLEMENTARY COURSE 1

MT3B01B23 - Vector Calculus, Differential Equations and Analytic Geometry

Time : 3 Hours **Maximum Marks : 80**

Part A

I. Answer any Ten questions. Each question carries 2 marks **(10x2=20)**

1. Describe circle of curvature and radius of curvature.
2. Find the derivative of the function $g(x, y, z) = x^2 + 2y^2 - 3z^2$ at the point (1, 1, 1) in the direction of **A** = **i+j+k** .
3. Compute the velocity vector at $t = \frac{1}{2}$ given $\boldsymbol{r}(t) = (t^2 + 1)\boldsymbol{i} + (2t - 1)\boldsymbol{j}$
4. Explain the relationship between path independence and potential function.
5. Evaluate the **k**-component of the curl for the vector field $\mathbf{F}(x, y) = (x^2 - y)\,\mathbf{i} + (xy - y^2)\,\mathbf{j}$
6. Compute the divergence of $\mathbf{F}(x, y) = (x^2 - y)\,\mathbf{i} + (xy - y^2)\,\mathbf{j}$.
7. Write the standard form of a First Order Linear Differential Equation.
8. Define an Exact Differential Equation.
9. Check whether the differential equation ydx-2(x+y)dy=0 is homogeneous or not.
10. List all Polar Coordinate representations for the point (-3, 0).
11. Sketch the parabola $x= -3y^2$. Also label the focus and directrix.
12. Find the equivalent cartesian equation of $r = 1 + 2r\cos\theta$

Part B

II. Answer any Six questions. Each question carries 5 marks **(6x5=30)**

13. (a). A glider is soaring upward along the helix $\mathbf{r}\,(t) = (\cos t)\,\mathbf{i} + (\sin t)\,\mathbf{j} + t\mathbf{k}$. How far does the glider travel along its path from $t = 0$ to $t = 2\pi$.

    (b) Show that if $\mathbf{u} = u_1\mathbf{i} + u_2\mathbf{j} + u_3\mathbf{k}$ is a unit vector, then the arc length parameter along the line $\mathbf{r}(t) = (x_0 + tu_1)\,\mathbf{i} + (y_0 + tu_2)\,\mathbf{j} + (z_0 + tu_3)\,\mathbf{k}$ from the point $P_0\,(x_0, y_0, z_0)$ where t=0 is t itself.
14. Calculate the length of the curve $\mathbf{r}(t) = (2 + t)\,\mathbf{i} - (t + 1)\,\mathbf{j} + t\mathbf{k},\ 0 \leq t \leq 3.$
15. Write the component test for Conservative Field. Determine whether the field $\mathbb{F} = (e^x\cos y + yz)\,\mathbf{i} + (xz - e^x\sin y)\,\mathbf{j} + (xy + z)\,\mathbf{k}$ is conservative or not.
16. Show that if $\mathbf{F} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$, then $\nabla \times \mathbf{F} = 0$.
17. Solve p(p+y)=x(x+y)
18. Examine whether the differential equation $(2x + y)dx - (x + 6y)dy = 0$ is exact.
19. Find the cartesian form of the polar equation $r = \frac{8}{1 - 2\cos\theta}$
20. What values of the constants a, b and c makes the ellipse $4x^2 + y^2 + ax + by + c = 0$ lie tangent to the x-axis at the origin and pass through the point (-1,2)? What is the eccentricity of the ellipse?

21. Find the polar equation of the circle $x^2 + 2x + y^2 = 0$. Sketch the circle.

Part C

III. Answer any Two questions. Each question carries 15 marks **(2x15=30)**

22. Compute $\mathbf{r}, \mathbf{T}, \mathbf{N}, \mathbf{B}$ and the equations for the osculating, normal and rectifying planes for the space curve $\mathbf{r}(t) = (cost)\,\mathbf{i} + (sint)\,\mathbf{j} + t\mathbf{k}$ at t=0.

23. Evaluate both the normal and tangential forms of Green's Theorem for the field $\mathbf{F}(x, y) = (x - y)\mathbf{i} + x\mathbf{j}$ and the region R bounded by the unit circle $\mathbf{r}(t) = (cost)\,\mathbf{i} + (sint)\,\mathbf{j}$, $0 \leq t \leq 2\pi$.

24. (a). Determine the general solution of $y= 2px-p^2$.

    (b). Solve $2dx-e^{y-x}dy =0$ by reducing it to an exact differential equation.

25. (a) Find the parametric equation of a cycloid.

    (b) Find the cartesian equivalent of $r = 1 + 2rcos\theta$