TM243843A

Reg. No :..............................

Name :..................................

MASTER'S DEGREE (C.S.S) EXAMINATION, NOVEMBER 2024

2023 ADMISSIONS REGULAR

SEMESTER III - CORE COURSE

MT3C15TM20 - Graph Theory

Time : 3 Hours **Maximum Weight : 30**

Part A

I. Answer any Eight questions. Each question carries 1 weight **(8x1=8)**

1. If G and H are isomorphic graphs then show that each pair of corresponding vertices of G and H has same degree.
2. Let G be a graph with n vertices and m edges and let each vertex of G is of degree k or k+1. Show that number of vertices of degree k in G is (k+1)n-2m.
3. For any three positive integers a,b,c with $a \leq b \leq c$, construct a simple graph with $\kappa = a,\ \lambda = b,\ \delta = c.$
4. Prove that a tree with at least 2 vertices contains at least two pendant vertices.
5. For a simple connected graph G if n(G) = m(G) show that G is unicyclic.
6. Define (a) Hamiltonian graph (b) Maximal non Hamiltonian graph (c) Hypo Hamiltonian graph.
7. If G is k-regular prove that $\chi(G) \geq n/n-k$.
8. Prove that for any simple planar graph G, $\delta(G) \leq 5$.
9. Show that the complement of a simple planar graph with 11 vertices is non-planar.
10. Show that every plane triangulation of order $n \geq 4$ is 3-connected.

Part B

II. Answer any Six questions. Each question carries 2 weight **(6x2=12)**

11. Define a connected graph. If G is a simple graph and $\delta \geq \frac{n-1}{2}$, show that G is connected.
12. Define tournament. Show that every tournament of order n has atmost one vertex v with $d^+(v) = n-1$.
13. Prove that an edge e is a cut edge of a graph G if and only if e does not belong to any cycle of G.
14. Write a short note on Dijkstra's Algorithm.
15. Let G be a simple graph with $n \geq 3$ vertices the G is Hamiltonian if and only if G + uv is Hamiltonian for all pair of non-adjacent vertices u and v in G with $d(u)+d(v) \geq n$.
16. For any simple graph G $2n \leq \chi + \chi^c \leq n+1 \ and \ n \leq \chi\chi^c \leq \frac{(n+1)^2}{2}$.
17. State and prove Euler's Formula.
18. Let A be a circulant matrix of order n with first row $(a_1, a_2, a_3 \dots a_n)$. Then prove that Sp (A) = { $a_1 + a_2 w + \dots a_n w^{n-1}$: w = nth root of unity} .

Part C

III. Answer any Two questions. Each question carries 5 weight **(2x5=10)**

19. a) If a bipartite graph G is regular, show that |X| = |Y|. b) Let G be any non-empty graph with at least two vertices then show that G is bipartite if and only if it has no odd cycles.
20. Prove that $\tau(K_n) = n^{n-2}$; $n \geq 2$.

21. For a connected graph G, prove that the following statements are equivalent. a) G is Eulerian. b) The degree of each vertex of G is an even positive integer. c) G is an edge disjoint union of cycles. Hence prove that Konigsberg Bridge problem has no solution.
22. a) Prove that every planar graph is 6 -vertex colorable. b) State and prove Heawood five-color theorem.