TB25034W

Reg. No:..............................

Name:..................................

B. Sc. DEGREE (C.B.C.S.S) EXAMINATION, MARCH 2025
(2016 and 2017 Admissions Supplementary)
SEMESTER VI – CORE COURSE (MATHEMATICS)
MT6B10B-COMPLEX ANALYSIS

Time: 3 Hours **Maximum Marks: 80**

PART A

I. Answer all questions. Each question carries 1 mark

1. Define hyperbolic tan function of a complex variable.
2. State Cauchy-Goursat theorem.
3. Write the nature of the singularity of the function $f(z) = e^{1/z}. 0 < |z| < \infty$.
4. Separate into real and imaginary parts $f(z) = 2z + 1$..
5. Find the limit of the sequence $z_n = \frac{1}{n^3} + i, n = 1,2 \dots$..
6. Write the Maclaurin's expansion of the function $\frac{1}{1-z}$, $|z|<1$.

(6x1=6)

PART B

II. Answer any seven questions. Each question carries 2 marks

7. Find the derivative of $(2z^2 + i)^5$.
8. If c is the positively oriented unit circle $|z| = 3$, then find the value of $\int_C \frac{1}{z-1}\, dz$.
9. Evaluate $\int_C \frac{z}{9-z^2} dz$, where C is the positively oriented circle $|z| = 2$.
10. State Fundamental theorem of algebra.
11. Find $\lim_{n\to\infty} -2 + i\frac{(-1)^n}{n^2}$
12. Find the residue of $\frac{1}{z+z^2}$ at z=0.
13. State Jordan's Lemma
14. Verify Cauchy Riemann equations for $f(z) = 3x + y + i(3y - x)$ ..
15. Show that $\frac{e^z}{z^2} = \frac{1}{z^2} + \frac{1}{z} + \frac{1}{21} + \frac{z}{31} + \cdots$
16. Expand the function $f(z) = \frac{1+2z^2}{z^3+z^5}$ in powers of z.

(7x2=14)

PART C

III. Answer any 5 questions. Each question carries 6 marks

17. Show that $e^{i\frac{\pi}{2}} = i$
18. State and prove Liouville's theorem.
19. Derive the Taylor series representation of sinz in powers of z.
20. Evaluate the integral $\int_C \frac{dz}{z(z-2)^4}$, where C i the positively oriented circle $|z-2| = 1$.
21. State and prove Cauchy's residue theorem.
22. Find the harmonic conjugate of$u(x, y) = y^3 - 3x^2y$.
23. Give Laurent series expansion in power of for the function$f(z) = \frac{1}{z^2(1-z)}$ and specify the regions in which the expansion is valid.
24. Evaluate$\int_c f(z)dz$ where $f(z) = \frac{z^2+1}{z^2-1}$ where $c: |z| = 2$.

(5x6=30)

PART D

IV. Answer any 2 questions. Each question carries 15 marks

25. State and prove Taylor's Theorem.
26. Show that$\int_{-\infty}^{\infty} \frac{\cos 3x}{(x^2+1)^2} dx = \frac{2\pi}{e^3}$.
27. If $f(z)$ is an analytic function inside and on a closed contour C described in the positive sense and z_0 is an interior pt of C ,then prove that $f^n(z_0) = \frac{n!}{2\pi i} \int_C \frac{f(z)}{(z-z_0)^{n+1}} dz$.
28. State and prove the necessary condition for a function to be analytic.

(2x15=30)