TB256183V

Reg. No :...............................

Name :.................................

BACHELOR'S DEGREE (C.B.C.S) EXAMINATION, MARCH 2025

2018, 2019, 2020, 2021 ADMISSIONS SUPPLEMENTARY

SEMESTER VI - CORE COURSE (MATHEMATICS)

MT6B10B18 - Complex Analysis

Time : 3 Hours **Maximum Marks : 80**

Part A

I. Answer any Ten questions. Each question carries 2 marks **(10x2=20)**

1. Sketch the region $0 \leq \arg z \leq \pi/4 (z \neq 0)$ and check whether it is a domain.
2. Is $|z-4| \geq |z|$ a bounded set?
3. Define an accumulation point.
4. Is the set of all points lying in the punctured plain $0 < |z| \leq 1$ an open or a closed set?
5. Is the set $1 < |z| < 2$ an open set? Is it connected?
6. If C is any closed curve lying in the open disk $|z| < 2$, then evaluate $\int_C \frac{ze^z}{(z^2+9)^5} dz$.
7. Prove the principle of deformation of paths.
8. Evaluate $\int_0^{1+i} z^2 dz$.
9. Show that when $z \neq 0$, $\frac{e^z}{z^3} = \frac{1}{z^2} + \frac{1}{z} + \frac{1}{2!} + \frac{z}{3!} + \frac{z^2}{4!} + \ldots$
10. Evaluate $\lim_{n \to \infty} -2 + i\frac{(-1)^n}{n^2}$ $(n = 1,2,3\ldots)$.
11. Define residue of a function f.
12. Evaluate the residue of $e^{\frac{1}{z^2}}$ at z=0.

Part B

II. Answer any Six questions. Each question carries 5 marks **(6x5=30)**

13. Show that $u = \frac{2xy}{(x^2+y^2)^2}$ and $v = \frac{x^2-y^2}{(x^2+y^2)^2}$ are harmonic functions.
14. Let $f(z) = e^y e^{ix}$. Express f(z) in the form u+iv. and using Cauchy Riemann equations show that f(z) is nowhere analytic.
15. Evaluate $\int_0^{\pi/4} e^{it} dt$.
16. Evaluate $\int_C \frac{dz}{z}$ where C is any positively oriented simple closed contour surrounding the origin.
17. Evaluate $\int_C \frac{z^2-1/3}{z^3-z} dz$ where C is $|z-1/2| = 1$

18. If w(t) is a piece wise continuous complex -valued function defined on an interval $a \leq t \leq b$, then $\left|\int_a^b w(t)dt\right| \leq \int_a^b |w(t)|dt.$

19. Show that when $z \neq 0$ $z^3\cosh\left(\frac{1}{z}\right) = \frac{z}{2} + z^3 + \sum_{n=1}^{\infty} \frac{1}{(2n+2)!} \frac{1}{z^{(2n-1)}}, 0 < |z| < \infty$

20. Find the order of the pole and its residue at z=2 of $\frac{z^2-2z+3}{z-2}$.

21. Evaluate the integral of f(z) over the positively oriented circle $|z| = 3$ where $f(z) = \frac{z^3(1-3z)}{(1+z)(1+2z^4)}$

Part C

III. Answer any Two questions. Each question carries 15 marks **(2x15=30)**

22. a. Suppose that $f(z) = u + iv$ and its conjugate $\overline{f(z)} = u - iv$ are analytic in a domain D, then show that $f(z)$ is constant in D.

    b. Let S be the open set consisting of all points z such that $|z| < 1$ or $|z-2| < 1$ .Is S connected?

    c. Sketch the closure of the following sets i) $Re\left(\frac{1}{z}\right) \leq \frac{1}{2}$ ii) $-\pi < arg z < \pi (z \neq 0)$.

23. a. State and prove Cauchy's extension formula.

    b. Evaluate $\int \frac{e^{2z}}{z^4} dz$, where the integral is to be evaluated over a positively oriented unit circle C: $|z| = 1$

24. State and prove Taylor's theorem.

25. Use residue to evaluate $\int_0^{\infty} \frac{x^2dx}{x^6+1}$.