TM243326Z

Reg. No :.....

Name :.....

MASTER'S DEGREE (C.S.S) EXAMINATION, NOVEMBER 2024

2023 ADMISSIONS REGULAR

SEMESTER III - CORE COURSE Applied Statistics and Data Analytics

ST3C11TM - Testing of Hypotheses

Time : 3 Hours **Maximum Weight : 30**

Part A

I. Answer any Eight questions. Each question carries 1 weight **(8x1=8)**

1. (a) What is Critical region. (b) Define a test function. Give example.
2. Explain the concepts of null hypothesis, alternate hypothesis and p value.
3. Show that most powerful test is a function of minimal sufficient statistic.
4. Define nuisance parameter. Explain how one can remove nuisance parameter?
5. Write a short note on Likelihood Ratio test.
6. Define Average Sample number.
7. Describe OC function in relation to SPRT.
8. Define parametric tests, give examples. How does the sample size affect the validity of parametric tests?
9. When would you choose the Wilcoxon rank-sum test (Mann-Whitney U test) instead of the Kruskal-Wallis test?
10. State the differences between one sample and two sample Kolmogorov test.

Part B

II. Answer any Six questions. Each question carries 2 weight **(6x2=12)**

11. If w is the critical region corresponding to the Most powerful test, show that power greater than or equal to significance level.
12. Show that one parameter exponential family possesses MLR property.
13. (a) Define unbiased test? (b) Explain how UMPU and LMPU are differ
14. Let $x_1, x_2, \ldots\ldots, x_n$ be a random sample from $N(\mu, \sigma^2)$ where σ^2 is known for testing $H_0 : \mu = \mu_0$ against $H_1 : \mu > \mu_0$. Find a UMA family of confidence sets for μ at level $1 - \alpha$.
15. Define SPRT, outlining its methodology and characteristics.
16. Given that following sequence of observations from normal distribution with σ = 15. Test the hypothesis H0: μ= 135 v/s H1: μ = 150 by means of SPRT with strength α = 0.01, β = 0.03. The observations are 121, 133, 144, 156, 104, 151, 155, 130, 160, 145, 120, 140, 125, 106, 123, 138, 108, 111, 118, 129, 123, 125, 149, 131. Draw the acceptance and rejection region.
17. Explain power and asymptotic relative efficiency of non- parametric tests.
18. Compare and contrast parametric and non-parametric tests, highlighting differences in assumptions, advantages, and limitations, along with situational examples.

Part C

III. Answer any Two questions. Each question carries 5 weight **(2x5=10)**

19. (a) State and prove the Neyman-Pearson fundamental lemma for Non randomized test. (b) State and prove the generalized Neyman-Pearson fundamental lemma for randomized test.
20. Obtain the level α likelihood ratio test for testing $H_0 : \mu \leq \mu_0$ against $H_1 : \mu > \mu_0$ based on a sample of n observations from $N(\mu, \sigma^2)$ where σ^2 is known.
21. State and prove Wald's identity. Hence deduce the O.C. function of a SPRT.
22. (a) Describe the Kruskal-Wallis test and its primary purpose in statistical analysis. (b) Explain the assumptions and conditions required for conducting a Kruskal-Wallis test. (c) Explain the steps to perform a Kruskal-Wallis test with a practical example.