TM241237T

Reg. No :...............................

Name :...............................

MASTER'S DEGREE (C.S.S) EXAMINATION, NOVEMBER 2024

2024 ADMISSIONS REGULAR

SEMESTER I - CORE COURSE MATHEMATICS

MT1C03TM20 - Real Analysis

Time : 3 Hours **Maximum Weight : 30**

Part A

I. Answer any Eight questions. Each question carries 1 weight **(8x1=8)**

1. If f is of bounded variation on [a,b], say $\sum |\Delta f_k| \leq M$ for all partition of [a,b], then prove that f is bounded on [a,b]. In fact $|f(x)| \leq |f(a)| + M \ \forall x \in [a,b]$.
2. Prove or disprove, "f(x)=[x] on [0,2] is of bounded variation"
3. Prove or Disprove Riemann Integral is a special case of Riemann-Stieltjes integral.
4. If $f \in R(\propto)$, then prove that $|\int_a^b f \, d \propto| \leq \int_a^b |f| \, d \propto$
5. Suppose $\{f_n\}$ is a sequence of functions defined on E and suppose $|f_n(x)| \leq M_n, x \in E, n = 1,2,3, \ldots$ Then prove that $\sum f_n$ converges uniformly on E if $\sum M_n$ converges.
6. If {fn} is a sequence of continuous function on E, and if fn→f uniformly on E then prove that f is continuous on E.
7. Prove or Disprove, "The convergent series of continuous function may have a discontinuous sum".
8. Define (i) Point wise bounded on E (ii) Equicontinuous on E
9. Prove that, $\lim_{x \to 0}(1+x)^{1/x} = e$
10. Prove or Disprove, "Every Convergent sequence contains a uniformly convergent subsequence".

Part B

II. Answer any Six questions. Each question carries 2 weight **(6x2=12)**

11. Let V be defined on [a,b] as follows :
    V(x) = $V_f(a,x)$ if a<x< b, V(a) = 0. Then prove that
    i. V is an increasing function on [a,b] and
    ii. V-f is an increasing function on [a,b].
12. Give an example for a continuous function which is not of Bounded variation and Justify your answer.
13. State and prove the necessary and sufficient condition for Riemann-Stieltjes integral.
14. If $f \epsilon R(\alpha)$ and $g \epsilon R(\alpha)$ on [a,b] then prove that
    a. $fg \epsilon R(\alpha)$
    b. $|f| \epsilon R(\alpha)$ and $\left| \int_a^b f d\alpha \right| \leq \int_a^b |f| \, d\alpha$
15. Show that the sequence of functions $\{f_n\}$ defined on E , converges uniformly on E if and only if for every, there exists an integer N such that $m, n \geq N, x \epsilon E$ implies $|f_n(x) - f_m(x)| \leq \epsilon$
16. Let {fn}, {gn} which converge uniformly on some set E, prove that {fngn} doesnot converge uniformly on E.

17. If K is compact, If $f_n \in C(K) for\ n = 1,2,3, \dots$ and if $\{f_n\}$ is point wise bounded and equicontinuous on K, then prove that

a. $\{f_n\}$ is uniformly bounded on K

b. $\{f_n\}$ contains a uniformly convergent subsequence.

18. Given a double sequence $\{a_{ij}\}$, i=1,2,3,.... , j=1,2,3,.... , suppose that $\sum_{j=1}^{\infty}|a_{ij}| = b_i , i = 1,2,3, \dots$ and $\sum b_i$ converges. Then show that $\sum_{i=1}^{\infty}\sum_{j=1}^{\infty} a_{ij} = \sum_{j=1}^{\infty}\sum_{i=1}^{\infty} a_{ij}$

Part C

III. Answer any Two questions. Each question carries 5 weight **(2x5=10)**

19. State and prove Jordan's Theorem.

20. State and prove the five properties of the Integrals.

21. Suppose $\{f_n\} \to f$ uniformly on a set E in a metric space. Let x be a limit point of E and suppose that $\lim_{t\to x} f_n(t) = A_n, n = 1,2,3\dots$ .Show that $\{A_n\}$ converges and $\lim_{t\to x} f(t) = \lim_{n\to\infty} A_n$.

22. Suppose $a_0, a_1, \dots, a_n$ are complex numbers, $n \geq 1, a_n \neq 0, P(z) = \sum_{n=0}^{\infty} a_k z^k$. Then show that P(z) = 0 for some complex number z.