TM241446J

Reg. No :...............................

Name :..................................

MASTER'S DEGREE (C.S.S) EXAMINATION, NOVEMBER 2024

2024 ADMISSIONS REGULAR

SEMESTER I - CORE COURSE MATHEMATICS

MT1C05TM20 - Ordinary Differential Equations

Time : 3 Hours **Maximum Weight : 30**

Part A

I. Answer any Eight questions. Each question carries 1 weight **(8x1=8)**

1. Write the standard form and normal form of Bessel's equation.
2. Show that any equation P(x)y''+ Q(x)y'+ R(x)y = 0 can be made self adjoint by multiplying through by $\frac{1}{P}e^{\int \frac{Q}{P}dx}$.
3. Using Rodrigue's Formula find the first four Legendre Polynomials
4. Show that $\frac{d}{dx}[xJ_1(x)] = xJ_0(x)$
5. Find the value of $J_{-3/2}(x)$
6. Check whether the function $-2x^2 + 3xy - y^2$ is positive dfinite, negetive definite or neither.
7. Find the general solution of the system $\begin{cases} \frac{dx}{dt} = x \\ \frac{dy}{dt} = -y \end{cases}$. Discuss the stability of critical points. Also sketch a few paths showing the direction of increasing t.
8. Determine the nature and stability of the critical point (0,0) on the linear autonomous system $\begin{cases} \frac{dx}{dt} = 2x \\ \frac{dy}{dt} = 3y \end{cases}$.
9. Write the first order initial value problem and its corresponding integral equation.
10. Show that $f(x, y) = y^{1/2}$ does not satisfy a Lipschitz condition on the rectangle $|x| \leq 1 \; and \; c \leq y \leq d, where \; 0 < c < d.$

Part B

II. Answer any Six questions. Each question carries 2 weight **(6x2=12)**

11. Show that the equation $P(x)y'' + Q(x)y' + R(x)y = 0$ is self adjoint if and only if $P'(x) = Q(x)$. Also write its self adjoint form.
12. Find the eigen values and eigen functions for the equation $y'' + \lambda y = 0; \; y(0) = 0, y(\pi) = 0$
13. Calculate the values of $P_2(x), P_3(x), P_4(x) \; and \; P_5(x)$ using the recursion formula $(n+1)P_{n+1}(x) = (2n+1)xP_n(x) - nP_{n-1}(x)$, assuming the value of $P_0(x) = 1 \; and \; P_1(x) = x$.
14. Prove that $J_n(x) = \frac{1}{\pi}\int_0^{\pi} cos(n\theta - xsin\theta)d\theta$

15. If $W(t)$ is the Wronskian of the two solutions $\begin{cases} x = x_1(t) \\ y = y_1(t) \end{cases}$ and $\begin{cases} x = x_2(t) \\ y = y_2(t) \end{cases}$ of the homogeneous system $\begin{cases} \frac{dx}{dt} = a_1(t)x + b_1(t)y \\ \frac{dy}{dt} = a_2(t)x + b_2(t)y \end{cases}$. Then Prove that $W(t)$ is either identically zero or nowhere zero on $[a,b]$.

16. Find the general solution of $\begin{cases} \frac{dx}{dt} = 4x - 3y \\ \frac{dy}{dt} = 8x - 6y \end{cases}$

17. Find the exact solution of the initial value problem $y' = 2x(1+y),\ y(0) = 0$. Let $y_0(x) = 1$, apply Picard's method to calculate $y_1(x), y_2(x), y_3(x), y_4(x)$.

18. Let P(x), Q(x) and R(x) be contonuous functions on an interval $a \le x \le b$. If x_0 is any point in the interval and y_0 and y_0' are any numbers whatever, prove that the initial value problem $\frac{d^2y}{dx^2} + P(x)\frac{dy}{dx} + Q(x)y = R(x);\ y(x_0) = y_0,\ y'(x_0) = y_0'$ has only one and only one solution on the interval $a \le x \le b$.

Part C

III. Answer any Two questions. Each question carries 5 weight **(2x5=10)**

19. a) State and prove Strum Comparison Theorem.

    b) Let $y(x)$ be the non trivial solution of Bessel's equation $x^2y'' - xy' + (x^2 - p^2)y = 0$ on positive x axis then prove that every interval of length π contains atmost one zero of $y(x)$ if $p = 1/2$.

20. a) Derive the Legendre Polynomials

    b) State and prove the orthogonality properties of Legendre Polynomials

21. a) Explain Volterra's Prey - Predator Equations

    b) Eliminate y from Volterra's Prey - Predator Equations and obtain the non linear second order equation satisfied by the function x(t).

22. Let $f(x,y) and\ {}^{\partial f}/_{\partial y}$ be continuous functions of x and y on a closed rectangle R with sides parallel to the axes. If (x_0, y_0) is any interior point of R then prove that there exists a number $h > 0$ with the property that the initial value problem $y' = f(x,y) \quad y(x_0) = y_0$ has one and only one solution $y = y(x)$ on the interval $|x - x_0| \le h$.