TM241951M

Reg. No :...............................

Name :.................................

MASTER'S DEGREE (C.S.S) EXAMINATION, NOVEMBER 2024

2024 ADMISSIONS REGULAR

SEMESTER I - CORE COURSE MATHEMATICS

MT1C04TM20 - Complex Analysis

Time : 3 Hours **Maximum Weight : 30**

Part A

I. Answer any Eight questions. Each question carries 1 weight **(8x1=8)**

1. Explain parabolic, elliptic, hyperbolic and loxodromic transformations.
2. Show that the sequence $\{f_n(z)\}$ converges uniformly on E if and only if to every $\epsilon > 0$ there exists a n_0 such that $| f_m(z) - f_n(z) | < \epsilon$ for all m, n $\geq n_0$ and for all z in E.
3. Prove that a general linear transformation $w = \frac{az+b}{cz+d}$ $(c \neq 0)$ is composed of a translation, an inversion, a rotation, and a homothetic transformation followed by another translation.
4. Prove: $\int_{-\gamma} f(z)dz = -\int_{\gamma} f(z)dz$
5. Compute the length of the circle $| z - a | = R$.
6. Define with examples (i) Pole (ii) Removable singularity
7. Describe algebraic order of f(z) at 'a' in the context of isolated singularity.
8. Explain (a) Homologous to zero (b) Locally Exact Differentials.
9. Find the residue of cot z at its poles.
10. How many roots does the equation $z^7 - 2z^5 + 6z^3 - z + 1 = 0$ have in the disk $|z| < 1$.

Part B

II. Answer any Six questions. Each question carries 2 weight **(6x2=12)**

11. State and prove Abel's Limit Theorem.
12. If $T_1(z) = (z+2) / (z+3)$, $T_2(z) = z / (z+1)$, find $T_1T_2(z)$, $T_2T_1(z)$, $T_1^{-1}T_2(z)$.
13. State and prove Cauchy's theorem in a disk.
14. Evaluate $\int_{|z|=2} \frac{dz}{z^2-1}$ for the positive sense of the circle.
15. State and prove Morera's theorem.
16. Show that a non constant analytic function maps open sets onto open sets.
17. State and prove Rouche's theorem.
18. Evaluate $\int_0^{\pi} \frac{d\theta}{a + cos\theta}; a > 1$

Part C

III. Answer any Two questions. Each question carries 5 weight **(2x5=10)**

19. (a) Discuss the uniform convergence of the series $\sum_1^{\infty} \frac{x}{n(1+nx^2)}$ for real values of x.
    (b) Show that the cross ratio of four points is invariant under a linear transformation.
20. (a) State and prove Cauchy's theorem in a disk with exceptional points.
    (b) Compute $\int_{|z|=1} |z-1||dz|$.

21. Suppose that $\varphi(\zeta)$ is continuous on the arc γ .Then show that the function $F_n(z)= \int_\gamma \frac{\varphi(\zeta)d\zeta}{(\zeta-z)^n}$ is analytic in each of the regions determined by γ and its derivative is $F_n'(z) = n\, F_{n+1}(z)$.

22. If p dx + q dy is locally exact in Ω, then $\int_\gamma p\, dx + q\, dy$ = 0 for every cycle $\gamma \sim 0$ in Ω. Prove.