TM241319Q

Reg. No :..............................

Name :..................................

MASTER'S DEGREE (C.S.S) EXAMINATION, NOVEMBER 2024

2024 ADMISSIONS REGULAR

SEMESTER I - CORE COURSE MATHEMATICS

MT1C02TM20 - Basic Topology

Time : 3 Hours

Maximum Weight : 30

Part A

I. Answer any Eight questions. Each question carries 1 weight **(8x1=8)**

1. Prove that a subset of a topological space is open if and only if it is a neighbourhood of each of its points.
2. Define a poset with an example.
3. State True or False: There is no convergent sequence in a trivial space. Justify
4. Prove that a set is clopen iff its boundary is empty.
5. Prove that, Let X be a topological space and Y be a subspace then a subset of Y is closed in Y iff it can be written as the intersection of Y and a closed set in X.
6. Prove that in a co-finite space any finite subset is closed.
7. Is the set of rational numbers connected? Justify.
8. Prove that regularity is a hereditary property.
9. Prove that a compact subset in a Hausdorff space is closed.
10. Prove that a topological space is T1 if and only if for any $x \in X$ the singleton set $\{x\}$ is closed.

Part B

II. Answer any Six questions. Each question carries 2 weight **(6x2=12)**

11. Show that in a co-countable space, if a sequence is convergent, then it will be an eventually constant sequence.
12. Prove that Metrisability is a hereditary property.
13. Prove that for a subset A of a space X, $\bar{A} = A \cup A'$
14. Define a dense subset of a topological space X. Prove that a subset A of a space X is dense in X if and only if for every non-empty open subset B of X , $A \cap B \neq \emptyset$.
15. Show that every regular Lindeloff space is normal.
16. Show that every Tychonoff space is regular.
17. Define a locally connected space. Prove that a space X is locally connected at a point $x \in X$ if and only if for every neighbourhood N of x, the component of N containing x is a neighbourhood of x.
18. Prove that a topological space X is regular if and only if for any $x \in X$ and any open set G containing x , there exists an open set H containing x such that $\overline{H} \subset G$

Part C

III. Answer any Two questions. Each question carries 5 weight **(2x5=10)**

19. Show that the intersection of topologies is again a topology and hence show that for any family of subsets of X, there exists a unique topology on X such that it is the smallest topology on X containing that family.
20. State and Prove the equivalent conditions for a function being Homeomorphism.
21. Prove that a subset of R is connected if it is an interval
22. Define a locally connected space. Prove that every quotient space of a locally connected space is locally connected. Also give an example of a space which is connected but not locally connected.